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Social Trading, Correlated Retail Investing and Non-Fundamental Speculation

David Russ*

Abstract

This paper shows that, in a setup à la Kyle (1985), correlated retail trading opens up new profit opportunities for professional investors at the expense of retail investors. Additionally, it demonstrates that market quality can benefit through higher market liquidity and higher price efficiency. Our results lend support to concerns that social trading via Finfluencers and stock message boards harms rather than benefits retail investors.

JEL classification: G12, G14

Key words: social trading, noise trading, non-fundamental information, strategic trading

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1 Introduction

“Social trading technology has made it increasingly simple for retail traders to trade in a coordinated manner and impact prices as a group.”

(Guan, 2023, p. 515)

The global rise of social media has changed financial markets significantly in several ways. Modern investors are increasingly attuned to the actions and sentiments of their peers, leading to the emergence of a novel investment approach known as *social trading*. This phenomenon is particularly prevalent among retail investors, who leverage social media platforms to exchange ideas and opinions before executing market orders. Stock message boards have become popular venues for such interactions, enabling retail investors to coordinate their trading activities. The impact of coordinated retail trading was vividly illustrated during the meme stock episodes involving companies like GameStop, AMC Entertainment, and BlackBerry.

Social trading is also fueled by financial influencers, or Finfluencers, who disseminate investment advice through platforms such as YouTube, X, and TikTok. As of 2024, Humphrey Yang, TikTok’s most renowned Finfluencer, boasts over 3.3 million followers and 49 million likes (see Arrowood, 2024). Empirical evidence provided by Hull and Qi (2024) and Keasey et al. (2025) highlights the influence of prominent Finfluencers on market prices. Notably, members of Generations Y and Z are particularly receptive to the advice of Finfluencers.¹ However, regulatory bodies such as the SEC and Germany’s BaFin caution that following Finfluencers poses financial risks for individual investors and contributes to market noise.^{2,3} Additionally, retail investors can engage in social trading by registering on dedicated social trading networks such as eToro or ZuluTrade. These trading networks allow retail investors to automatically replicate the trades of other registered investors, which is referred to as so-called “copy trading”.

Simultaneously, there is substantial empirical and anecdotal evidence indicating that professional investors such as hedge funds, banks, and pension funds actively monitor

¹See <https://www.creditkarma.com/about/commentary/gen-z-turns-to-tiktok-and-instagram-for-financial-advice-and-actually-takes-it-study-finds>.

²See <https://www.sec.gov/files/approved-finfluencer-recommendations-20241210.pdf>.

³See https://www.bafin.de/EN/Verbraucher/Finanzbetrug/Anlagebetrug/Social_Media/social_media_artikel_en.html.

retail investor behavior in financial markets. These professional traders employ advanced big data analytics and artificial intelligence to scrutinize social media content and gauge retail investor sentiment.⁴ By capitalizing on insights derived from retail investing patterns, professional investors aim to raise their profits. This trading strategy is called *non-fundamental speculation* (see, e.g., Madrigal, 1996).

In an environment characterized by non-fundamental speculation, this paper explores the impact of social trading on retail investors' and professional investors' expected wealth and key market quality measures, viz., market liquidity and price efficiency. We depart from Kyle's (1985) seminal framework and introduce N -dimensional retail trader demand with positive correlation to account for social trading. As standard in the literature, we model retail traders as noise traders (see, e.g., Peress and Schmidt 2019, 2021; Barber et al., 2022; Eaton et al., 2022). We consider both a static variant as well as a dynamic variant with two trading dates. In the baseline model of the dynamic variant, we restrict our attention to correlation between noise traders *within* trading dates and ignore correlation *across* trading dates. This assumption mostly relates to the form of social trading that retail investors engage in on dedicated social trading networks, which allow them to automatically copy the trades of other investors. In an extension of the dynamic variant, we additionally introduce correlation across trading dates. By that, we pay attention to the fact that other forms of social trading such as following conversations on stock message boards or taking advice from famous Finfluencers lead to noise trading being correlated not only within but also across trading dates.

To account for non-fundamental speculation, we assume different rational, non-fundamental speculators who observe different noise components. In the baseline model, we assume that each non-fundamental speculator observes exactly one noise component, as in Roell (1990). In an extension, we allow non-fundamental speculators to observe more than one noise component simultaneously.

Our central result is that social trading *worsens* retail investors' expected losses to the benefit of professional investors. This occurs in both the static and dynamic versions of our model. Social trading c.p. increases the price impact of noise trading. Since their trading behavior is uncorrelated with fundamentals, the additional price impact raises their expected losses to the benefit of professional investors.

⁴See <https://www.wsj.com/articles/fund-managers-pay-attention-to-retail-day-traders-11642132135>.

In the static version of our model, social trading leaves price efficiency unaffected. This outcome is similar to the irrelevance result of Rochet and Vila (1994), who show that in a static setup, non-fundamental speculation leaves price efficiency unchanged. This insight carries over to correlation in noise trading. Perhaps surprisingly, in the static model, social trading harms market liquidity. In a setup with non-fundamental speculation, social trading exerts two counteracting effects on market liquidity. On the one hand, it injects an additional source of noise into aggregate order flow, attenuating market makers' adverse selection problem. This raises market liquidity. On the other hand, social trading makes non-fundamental speculators trade more aggressively against noise trader demand, reducing the noise in aggregate order flow and lowering market liquidity. The model reveals that the latter effect dominates the former one, and social trading is detrimental to market liquidity. In the dynamic model, however, social trading can be conducive to market liquidity at both trading dates if comovement in retail trader demand is sufficiently weak. Interestingly, price efficiency at both trading dates is now increasing in the intensity of social trading.

Social trading has furthermore two important implications for the nature of non-fundamental speculation in a setup à la Kyle (1985). First, non-fundamental speculators trade more aggressively against noise if noise traders engage in social trading. In the presence of social trading, speculators know that a rise in their observed noise trader component translates into a rise in the unobserved noise components, which intensifies the initial impact of the rise in noise trading on the price. This additional effect on the price leads profit-maximizing speculators to offset more of the noise trading they observe. Second, social trading induces substitutability in non-fundamental speculators' trading intensities against noise. That is, due to social trading, the trading intensities against noise trader demand are negatively connected. Rational speculators take the other speculators' effect on the price through trading against the unobserved noise trader components into account. If the observed part of noise trader demand rises, each speculator knows that the other speculators will raise their trading intensity against noise, which partly counteracts the effect of noise trading on the price already. This leads each speculator to absorb less of the noise trader demand she observes. In other words, a non-fundamental speculator would counteract more of her observed noise trader demand if the other speculators did not exist.

Our paper is connected to the literature on non-fundamental speculation within Kyle's (1985) seminal framework. In a paper closely related to ours, Roell (1990) introduces a static model with non-fundamental speculators who observe one of the several noise components

each. In contrast to our paper, the different noise trader demands are uncorrelated. Roell (1990) uncovers that non-fundamental speculation raises noise traders’ expected wealth as long as there is residual noise unobserved by speculators. Rochet and Vila (1994) adds to Roell’s (1990) result by demonstrating that expected profits of rational and noise traders are the same if all noise is observed and unobserved, respectively. In our setup, social trading unambiguously raises noise traders’ expected losses. In line with Rochet and Vila (1994), their losses would be the same if non-fundamental speculation did not exist. Our model, however, reveals an additional new result. We demonstrate that noise traders’ expected losses can be higher with unobserved residual noise and social trading than without both non-fundamental speculators and social trading. That is, in the presence of social trading, introducing non-fundamental speculation with residual noise may not undo the negative impact of social trading on retail traders’ expected wealth.

Madrigal (1996) and Yang and Zhu (2017) develop a two-trading-date model with a non-fundamental speculator and show that non-fundamental speculation can lead to lower market liquidity and lower price efficiency. Foucault and Lescourret (2003) explore the impact of information sharing between a fundamental and a non-fundamental speculator. They reveal that information sharing reduces noise traders’ expected losses. Cheynel and Levine (2012) consider a model where an analyst sells non-fundamental information to market participants. Demarquette (2016) endogenizes fundamental and non-fundamental information acquisition. In his setup, the possibility to acquire non-fundamental information lowers the welfare of noise traders. In a dynamic setup, Bernhardt and Taub (2008) assume a non-fundamental speculator who observes both contemporaneous and future noise trading (which are uncorrelated). The ensuing front-running behavior raises speculator’s expected profits at the expense of noise traders. In a similar vein, Sadzik and Woolnough (2021) model noise trading to be persistent over time. They prove that a non-fundamental speculator with knowledge about the persistent component of noise trading can destabilize markets. Waters et al. (2024) obtain similar results in a dynamic model where noise trading follows a moving average process. In contrast to all listed contributions, our model assumes *within-period* correlation in noise trader demand. This yields the aforementioned new insights.

Our paper is moreover related to contributions that explore the impact of non-fundamental information in the canonical framework developed by Grossman and Stiglitz (1980). This framework differs in two important aspects from that used in this paper. First, agents are assumed to be risk-averse instead of risk-neutral. Second, the market is assumed to

be competitive, which means that agents condition their demand on the contemporaneous price. In such a setting, agents use information about current noise trading to extract more precise information about fundamentals out of the market price. Ganguli and Yang (2009) and Manzano and Vives (2011) show that non-fundamental information can lead to multiple equilibria with different market properties. Russ (2022) demonstrates that trading against different types of noise is a complement, rather than a substitute as in our model. More aggressive trading against noise by one rational agent makes the price a more precise signal about fundamentals for all other rational agents. This makes them trade more aggressively on this very price signal, which entails more aggressive trading against the noise component they observe. Cespa and Vives (2012, 2015) show that persistent noise trading can lead to multiple equilibria, whereas one equilibrium is characterized by high price efficiency. Marmora and Rytchkov (2018) and Farboodi and Veldkamp (2020) reveal that acquiring non-fundamental information raises price efficiency in a static and a dynamic setup, respectively. In a dynamic model, Arnold and Russ (2024), by contrast, point to the possibility of information about future noise trading harming price efficiency.

The remainder of this paper is organized as follows. Section 2 analyzes the static version of the model with social trading. Section 3 explores the dynamic model with two trading dates. Section 4 provides extensions to the baseline models by allowing rational speculators to observe more than one noise component simultaneously and by adding intertemporal correlation among noise trading, respectively. Section 5 concludes. Model proofs are collected in the Appendix.

2 Static model

This section develops a static setup, where equilibrium can be derived in closed form and the relevant economic effects can be clearly identified. It demonstrates that social trading unambiguously worsens retail traders' expected losses to the benefit of professional investors. At the same time, it deteriorates market liquidity while leaving price efficiency unaffected.

Model

Our model is a variant of the seminal model of Kyle (1985). Consider a static financial market in which a risky asset is traded at market price p . The liquidation value of the risky asset one period ahead is $v \sim N(0, \sigma_v^2)$. The market is populated by $N \geq 2$ rational,

risk-neutral speculators and N noise traders. The exogenous noise trader demand u_i ($i = 1, 2, \dots, N$) $\sim N(0, \sigma_u^2)$ is independent of v . Since noise traders engage in social trading via following famous Finfluencers' advices or conversations in stock message boards, their demands are correlated. More specifically, we assume $Cov(u_i, u_{i'}) = c$, with $0 < c \leq \sigma_u^2$ ($i, i' = 1, 2, \dots, N, i \neq i'$). Rational speculators behave strategically, i.e., they take the impact of their own market order on the price into account. All rational speculators perfectly observe the risky asset's fundamental value v and submit market orders d_i ($i = 1, 2, \dots, N$).⁵ In addition, each rational speculator observes one of the N noise trader demands. The market is made by a set of competitive, rational market makers who earn zero profit in expectations and set the market price equal to their expectations about the risky asset's liquidation value based on the observed aggregate order flow. Due to noise trading, the market makers are faced with a signal extraction problem, as they do not know whether a large order flow is linked to strong fundamentals or to high noise trading.

Equilibrium

The equilibrium concept we resort to is the standard one used in the relevant strand of the literature. In equilibrium, rational speculators maximize expected profit based on their information, and the market makers set the price equal to their conditional expectations about the asset's fundamentals.

Definition (static equilibrium): A price function $p(v, \sum_{i=1}^N u_i)$ and market orders d_i ($i = 1, 2, \dots, N$) are an equilibrium if (a) $p = E(v | w = \sum_{i=1}^N d_i + \sum_{i=1}^N u_i)$ and (b) d_i maximizes $E[(v - p)d_i | v, u_i]$ for $i = 1, 2, \dots, N$.

We only consider linear equilibria, meaning that the price p is assumed to be linear in the market participants' order flow:

$$p = \lambda \left(\sum_{i=1}^N d_i + \sum_{i=1}^N u_i \right), \quad (1)$$

where $\lambda = \frac{Cov(v, w)}{Var(w)}$. More fundamentally informed trading, expressed by a higher covariance between v and w , raises the adverse selection problem for market makers. To protect themselves against the risk of trading against informed investors, they set a price that is more sensitive to changes in order flow (i.e., λ rises).

⁵A noisy signal about v instead of a noiseless one would leave all derived results unchanged.

Analogously, we assume rational speculators' market orders to be linear in the state variables:

$$d_i = \alpha_i v - \beta_i u_i, \quad (2)$$

for constants α_i and β_i ($i = 1, 2, \dots, N$). Solving rational agents' profit maximization problems and determining the price set by the market makers yields:

Proposition 1: *There exists a unique linear equilibrium, in which*

$$\begin{aligned} \alpha_i &= \frac{1}{(N+1)\lambda}, \\ \beta_i &= \frac{(N-1)c + \sigma_u^2}{(N-1)c + 2\sigma_u^2}, \quad \text{for } i = 1, 2, \dots, N, \\ \lambda &= \frac{[c(N-1) + 2\sigma_u^2]\sigma_v}{(N+1)\sigma_u^2\sqrt{[c(N-1) + \sigma_u^2]}}. \end{aligned}$$

The proof can be found in Appendix A. Proposition 1 yields a unique, closed-form solution for the coefficient array $(\alpha_i, \beta_i, \lambda)$. As one would expect, rational speculators' trading strategies are symmetric, i.e., the coefficients α_i and β_i do not depend on i . We see that $0.5 < \beta_i < 1$. By (2), the non-fundamental speculators trade against noise trader demand (i.e., $\partial d_i / \partial u_i < 0$), mitigating its impact on the price. Rational speculators know that the demand stemming from noise traders affects the market price, while leaving fundamentals unchanged. Thus, a higher (resp., lower) noise trader demand exerts upward (resp., downward) pressure on the market price only. The ensuing higher (resp., lower) price induces rational speculators to demand a smaller (resp., higher) share of the risky asset, partly offsetting the impact of noise trading on the market price. The fact that β_i is bounded between 0.5 and 1 means that rational speculators offset more than half of the impact of noise trading, but not all of it. If they counteracted all of noise traders' influence, aggregate order flow would reveal the true value of v to the market makers. This would lead them to set the market price equal to fundamentals, eliminating all of rational speculators' profit opportunities. The strategically behaving rational speculators take this fact into account and therefore only partially offset noise traders' impact.

Notably, trading intensity β_i increases with c (i.e., $\partial \beta_i / \partial c > 0$). That is, with comovement in retail investing, non-fundamental speculators' trading intensities are higher than in standard settings without comovement (cf. Roell, 1990; Demarquette, 2016). In the presence of social trading, speculator i knows that a rise in u_i translates into a rise in the other, unobserved

noise components, raising the impact of an increase in u_i on the market price. This additional impact on the price makes profit-maximizing speculator i respond more strongly to changes in her observed noise component (i.e., β_i increases), offsetting more noise originating from u_i .

Before we turn to assessing the impact of social trading on other important market properties, we are interested in rational speculators' strategic interaction when trading against noise. That is, we are interested in how the trading intensities β_i and $\beta_{i'}$ ($i, i' = 1, 2, \dots, N, i \neq i'$) are connected. From (A.1) in Appendix A,

$$\beta_i = \frac{(N-1)c + \sigma_u^2 - \sum_{i' \neq i}^N c \beta_{i'}}{2\sigma_u^2}. \quad (3)$$

Inspecting (3), the next proposition follows immediately:

Proposition 2: *Trading against different types of noise is a substitute (i.e., $\partial\beta_i/\partial\beta_{i'} < 0$).*

According to Proposition 2, a setup with strategic speculation à la Kyle (1985) is characterized by substitutability in trading against different types of noise.⁶ The economic intuition goes as follows: In the presence of social trading, speculator i takes speculator i' 's impact on the market price through trading against noise trader demand $u_{i'}$ into account. If noise trader demand u_i rises, speculator i knows that speculator i' will partly offset the impact of the ensuing rise in $u_{i'}$ already. This lower overall price impact of u_i , in turn, makes speculator i absorb less of noise trader demand than in the absence of speculator i' . Note that if $c = 0$, the connection between the trading intensities vanishes. In the special case without social trading, speculator i knows that a rise in u_i does not affect the trading behavior of speculator i' . This makes trading intensity β_i completely insensitive to changes in $\beta_{i'}$.

Market Properties

Having derived equilibrium, we proceed to evaluate the effect of social trading on important market properties, viz., (i) market depth, (ii) price efficiency, (iii) noise traders' ex-ante expected losses $E(\pi_{NT})$, and (iv) rational speculators' ex-ante expected profits $E(\pi_{SP})$. We get:

⁶This contrasts sharply with the canonical framework of Grossman and Stiglitz (1980), where trading against different types of noise is unequivocally a complement (see Russ, 2022).

$$\text{market depth} \equiv \frac{1}{\lambda} = \frac{(N+1)\sigma_u^2 \sqrt{[c(N-1) + \sigma_u^2]}}{[c(N-1) + 2\sigma_u^2]\sigma_v}, \quad (4)$$

$$\text{price efficiency} \equiv \frac{1}{\text{Var}(v|p)} = (N+1)/\sigma_v^2, \quad (5)$$

$$E(\pi_{NT}) = NE[(v-p)u_i] = -\frac{N}{(N+1)}\sigma_v \sqrt{[c(N-1) + \sigma_u^2]}, \quad (6)$$

$$E(\pi_{SP}) = NE[(v-p)d_i] = \frac{N}{(N+1)}\sigma_v \sqrt{[c(N-1) + \sigma_u^2]}. \quad (7)$$

By (4) to (7), the next proposition follows immediately:

Proposition 3: *In equilibrium, the market exhibits the following properties:*

- a) market depth depends negatively on social trading: $\partial(1/\lambda)/\partial c < 0$,*
- b) price efficiency is independent of social trading: $\partial \text{Var}^{-1}(v|p)/\partial c = 0$,*
- c) noise traders' expected losses are increasing in c : $\partial E(\pi_{NT})/\partial c < 0$,*
- d) rational speculators' expected profits are rising in c : $\partial E(\pi_{SP})/\partial c > 0$.*

Parts (a) reveals that social trading worsens market depth. Comovement in noise trader demand exhibits two counteracting effects on the intensity of adverse selection market makers are confronted with. On the one hand, the direct link between u_i and $u_{i'}$ c.p. injects additional noise into aggregate order flow, alleviating the adverse selection problem and thereby increasing market depth. On the other hand, social trading makes speculators trade more aggressively against noise, mitigating its impact. This, in turn, makes aggregate order flow appear more informed to market makers, inducing them to lower market depth. The result in Proposition 3 reveals that the latter, detrimental effect dominates the former one. Thus, in the presence of social trading and non-fundamental speculation, market makers are faced with more serious adverse selection. This raises the sensitivity of the market price to changes in aggregate order flow, lowering market depth.

According to part (b), in the static setup, price efficiency is independent of c as well as of non-fundamental speculators' trading intensities β_i . This result is similar to the irrelevance result of Rochet and Vila (1994). In the presence of social trading and non-fundamental speculation, aggregate order flow appears more informed to market makers. This makes them steepen the price schedule to protect themselves against more severe adverse selection (i.e., λ rises). Rational speculators anticipate market makers' behavior and scale down their

trading intensities on fundamentals, α_i , proportionally. In aggregate, price efficiency remains unchanged. To illustrate this result in a more formal way, we can express price efficiency as

$$\text{Var}^{-1}(v | p) = \sigma_v^{-2} + \frac{N\alpha_i^2[c(N-1) + 2\sigma_u^2]^2}{\sigma_u^4[c(N-1) + \sigma_u^2]}.$$

Consequently,

$$\frac{d\text{Var}^{-1}(v | p)}{dc} = \underbrace{\frac{\partial \text{Var}^{-1}(v | p)}{\partial c}}_{>0} + \underbrace{\frac{\partial \text{Var}^{-1}(v | p)}{\partial \alpha_i} \frac{d\alpha_i}{dc}}_{<0} = 0.$$

The first term in the above derivative represents the direct, positive impact of c on price efficiency. As social trading intensifies, rational speculators trade more aggressively against noise. This makes aggregate order flow more informed, benefiting price efficiency. However, as a consequence of a more informed order flow, market makers set a price that is more sensitive to changes in order flow. This, in turn, makes rational speculators trade less aggressively on fundamental information, harming price efficiency, which is represented by the second, negative term in the above derivative. More specifically, the negative effect operating through less intense fundamental speculation exactly offsets the positive impact stemming from more aggressive non-fundamental speculation. This explains why, in aggregate, price efficiency is insensitive to changes in c .

Part (c) of the proposition reveals the negative impact of social trading on retail traders' expected wealth. Their expected losses are increasing in c . This outcome uncovers that following Finfluencers, conversations in stock message boards, and other social media sources *harms* rather than benefits retail traders. Retail traders would be better off if they did not pay any attention to social media (i.e., $c = 0$). The comovement of their market orders opens up profit opportunities for rational investors (see part (d)). Thus, parts (c) and (d) indicate that social trading creates new profit opportunities for professional investors on the back of retail traders.

To understand in more detail why social trading worsens retail investors' expected losses, it is instructive to rewrite $E(\pi_{NT})$ as follows:

$$E(\pi_{NT}) = -NCov(p, u_i) = -N\lambda[(1 - \beta_i)\sigma_u^2 + (N - 1)(1 - \beta_i)c].$$

We see that the parameter c exerts three effects on $E(\pi_{NT})$, two of which are negative. First, λ

raises, as market makers' adverse selection risk intensifies. This makes the price more sensitive to changes in noise trader demand. Second, as u_i increases, $u_{i'}$ rises too, extending the initial impact on the market price. This is represented by the second term in square brackets. However, there is also one positive effect, since speculators trade more aggressively against noise, mitigating its impact on the price. Nevertheless, the two negative effects dominate the positive one, making noise traders' demand vary more strongly with the price. Consequently, their expected wealth is falling in c .

Comparison with Roell (1990) and Rochet and Vila (1994). The fact that correlation among different noise traders' market orders lowers their expected wealth adds a new insight to the results of Roell (1990) and Rochet and Vila (1994). In a setup closely related to ours, Roell (1990) analyzes a Kyle model with several rational, non-fundamental speculators, each of whom observes a single noise component. In sharp contrast to our setup, the different noise components are uncorrelated. Roell (1990) demonstrates that non-fundamental speculation *raises* noise traders' expected wealth as long as there is residual noise (i.e., some noise trading that is unobserved by the speculators). Thus, in our model, holding the parameter c constant, noise traders' losses would be lower if, e.g., $\beta_N = 0$.

Rochet and Vila (1994) consider a variant with a single non-fundamental speculator who observes the single noise component of the model. They prove that noise traders' expected wealth is the same as in the case without a non-fundamental speculator. That is, without residual noise, noise traders neither suffer nor benefit from non-fundamental speculation. In our model, for any fixed value of c , this means that noise traders' expected losses would stay the same if no information about noise trading existed (i.e., $\beta_i = 0$ for $i = 1, 2, \dots, N$).

While the two outcomes regarding noise trader welfare of Roell (1990) and Rochet and Vila (1994) are well known, our model adds another new result to the relevant literature. It establishes that noise traders' expected losses can be *higher* in the presence of $N - 1$ non-fundamental speculators and social trading than in the absence of both non-fundamental speculators and social trading:

Proposition 4: *If $c > \bar{c}$,*

$$E(\pi_{NT})[\beta_i(i = 1, 2, \dots, N - 1) > 0, \beta_N = 0, c > 0] < E(\pi_{NT})[\beta_i(i = 1, 2, \dots, N) = c = 0].$$

The proof as well as the solution for $\bar{c}$ can be found in Appendix A. According to Proposition 4, although $N - 1$ non-fundamental speculators c.p. attenuate N noise traders' expected losses, these losses can be higher for $c > 0$ than in the case without non-fundamental

	$c > 0$	$c = 0$
$\beta_i > 0$ or $\beta_i = 0$ ($i = 1, 2, \dots, N$)	worst outcome	second-best outcome if $c > \bar{c}$
$\beta_i > 0$ ($i = 1, 2, \dots, N - 1$), $\beta_N = 0$	second-worst outcome if $c > \bar{c}$	best outcome

Table 1: Comparison of noise traders' expected losses

information and comovement in noise trader demand. Thus, social trading can be detrimental to noise traders even if $N - 1$ non-fundamental speculators exist, and noise traders would be better off if non-fundamental information and comovement were absent. More specifically, this case occurs for values of c large enough, as noise trader losses are still increasing in c for $\beta_i (i = 1, 2, \dots, N - 1) > 0, \beta_N = 0$ (see Appendix A). Hence, there exists a unique value of c , $\bar{c}$ say, above which the condition in the proposition is satisfied.

Table 1 summarizes how different combinations of social trading and non-fundamental speculation affect noise traders' expected losses. Noise traders are best off if there exists residual noise (i.e., $\beta_N = 0$) and social trading is absent (i.e., $c = 0$). The unambiguously worst outcome for them materializes whenever there is social trading in an environment without non-fundamental speculation or without residual noise. In line with Proposition 4, if social trading is strong enough (i.e., $c > \bar{c}$), noise traders would be better off in an environment without non-fundamental speculation and without social trading than in an environment with both residual noise and social trading.

3 Dynamic model

This section develops a dynamic two-period setup and demonstrates that social trading still harms retail investors. While the overall complexity of the model precludes a full analytical characterization of equilibrium, a calibration of the model using parameter values derived from the relevant literature reveals that noise traders' expected losses still decline in the intensity of social trading. Interestingly, market quality can now benefit. Market liquidity at both trading dates rises for sufficiently weak social trading. Furthermore, price efficiency at both the early and late dates unequivocally benefits from social trading. At the end of the section, we provide an empirical proxy for retail investors' losses due to social trading.

Model

There are two trading dates t ($= 1, 2$) before the asset pays off its fundamental value v . At each trading date, there are $N \geq 2$ noise traders with exogenous demand $u_{ti} \sim N(0, \sigma_u^2)$ for $i = 1, 2, \dots, N$. As in Madrigal (1996), Bernhardt and Taub (2008), and Yang and Zhu (2020), we assume noise trading to be uncorrelated across time (i.e., $Cov(u_{1i}, u_{2i'}) = 0$ for $i, i' = 1, 2, \dots, N$).⁷ Within periods, we still assume noise trading to be correlated. More specifically, $Cov(u_{1i}, u_{1i'}) = Cov(u_{2i}, u_{2i'}) = c$ ($i, i' = 1, 2, \dots, N, i \neq i'$), with $0 < c \leq \sigma_u^2$.⁸ As in the static model, there are N rational speculators who submit market orders d_{ti} ($t = 1, 2; i = 1, 2, \dots, N$). Each speculator observes v as well as one noise trader demand component at each trading date. The market is made by competitive market makers who observe the market participants' order flows and set prices p_1 and p_2 equal to their conditional expectations about fundamentals v .

Equilibrium

In equilibrium, each rational speculator chooses her market orders at both trading dates to maximize expected lifetime profits. Additionally, the market makers use date-1 order flow to set market price p_1 and date-1 as well as date-2 order flows to determine p_2 .

Definition (dynamic equilibrium): Price functions $p_1(v, \sum_{i=1}^N u_{1i})$ and $p_2(v, \sum_{i=1}^N u_{1i}, \sum_{i=1}^N u_{2i})$ and market orders d_{1i}, d_{2i} ($i = 1, 2, \dots, N$) are an equilibrium if (a) $p_1 = E(v | w_1 = \sum_{i=1}^N d_{1i} + \sum_{i=1}^N u_{1i})$, (b) $p_2 = E(v | w_1, w_2 = \sum_{i=1}^N d_{2i} + \sum_{i=1}^N u_{2i})$, and (c) d_{1i}, d_{2i} maximize $E\{(v - p_1)d_{1i} + E[(v - p_2)d_{2i} | v, u_{1i}, u_{2i}, p_1] | v, u_{1i}\}$ for $i = 1, 2, \dots, N$.

Analogously to the static model, we restrict our attention to linear equilibria. Prices at both trading dates are thus assumed to be linear in order flows:

$$p_1 = \lambda_1 \left(\sum_{i=1}^n d_{1i} + \sum_{i=1}^n u_{1i} \right), \quad (8)$$

$$p_2 = p_1 + \lambda_2 \left(\sum_{i=1}^n d_{2i} + \sum_{i=1}^n u_{2i} \right), \quad (9)$$

where $\lambda_1 = \frac{Cov(v, w_1)}{Var(w_1)}$ and $\lambda_2 = \frac{Cov(v, w_2 | w_1)}{Var(w_2 | w_1)}$. As before, we model rational speculators' market

⁷The case of persistent noise trading in a dynamic setup à la Kyle (1985) is, e.g., treated in Sadzik and Woolnough (2021) and Waters et al. (2024).

⁸Different covariances across time would unnecessarily complicate the analysis while leaving all derived results unchanged.

orders as linear functions of the random variables of the model:

$$d_{1i} = \alpha_{1i}v - \beta_{1i}u_{1i}, \quad (10)$$

$$d_{2i} = \alpha_{2i}v - \beta_{2i}u_{2i} - e_{2i}p_1, \quad (11)$$

for constants α_{ti} , β_{ti} , and e_{2i} ($t = 1, 2; i = 1, 2, \dots, N$). Solving the model backwards yields the following conditions in equilibrium:

Proposition 5: *In a linear dynamic equilibrium, it holds that*

$$\begin{aligned} \alpha_{2i} &= \frac{1}{(N+1)\lambda_2}, \\ \beta_{2i} &= \frac{(N-1)c + \sigma_u^2}{(N-1)c + 2\sigma_u^2}, \\ e_{2i} &= \frac{1}{(N+1)\lambda_2}, \\ \alpha_{1i} &= \frac{2\lambda_1 - (N+1)^2\lambda_2}{2N\lambda_1^2 - (N+1)^3\lambda_1\lambda_2}, \\ \beta_{1i} &= \frac{[2\lambda_1 - (N+1)^2\lambda_2][c(N-1) + \sigma_u^2]}{c(N-1)[2\lambda_1 - (N+1)^2\lambda_2] + 2[\lambda_1 - (N+1)^2\lambda_2]\sigma_u^2}, \end{aligned}$$

for $i = 1, 2, \dots, N$. The values of λ_1 and λ_2 are obtained by solving

$$\lambda_1 = \frac{N\alpha_{1i}\sigma_v^2}{(N\alpha_{1i})^2\sigma_v^2 + N(1 - \beta_{1i})^2\sigma_u^2 + (N-1)N(1 - \beta_{1i})^2c}, \quad (12)$$

$$\lambda_2 = \frac{[N\alpha_{1i}\text{Cov}(w_1, w_2) - N\bar{\alpha}_{2i}\text{Var}(w_1)]\sigma_v^2}{\text{Cov}^2(w_1, w_2) - \text{Var}(w_1)\text{Var}(w_2)}. \quad (13)$$

The proof as well as the expressions for $\bar{\alpha}_{2i}$, $\text{Cov}(w_1, w_2)$, $\text{Var}(w_1)$, and $\text{Var}(w_2)$ can be found in Appendix A. The number of equilibria is pinned down by the solutions for the two-equation system in (12) and (13). As usual in dynamic Kyle models, the high non-linearity of the system precludes an analytical solution of the model. To illustrate the equilibrium properties, we resort to a numerical simulation with reasonable values for the model parameters. More specifically, we calibrate the model resorting to parameter values used in the relevant literature on dynamic models in the Kyle framework. As in Yang and Zhu (2020), we set the variance of fundamentals σ_v^2 to 0.00036 and the variance of retail trader demand σ_u^2 to 1. This leads to a range for c of $0 < c \leq 1$. Additionally, we let $N = 2$.

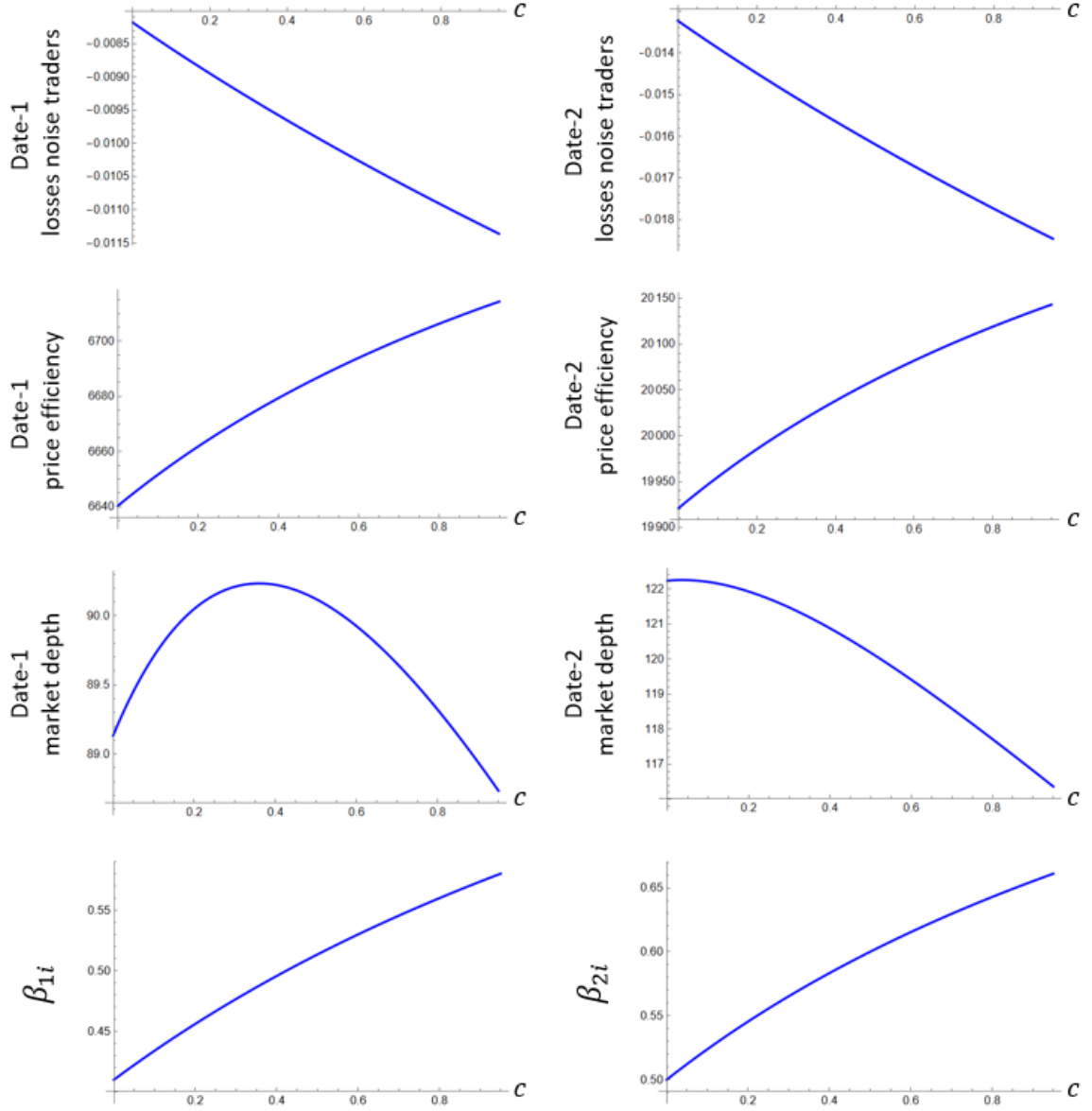


Figure 1: **Comparative statics in the dynamic model (baseline case).** Parameters: $\sigma_v^2 = 0.00036$, $\sigma_u^2 = 1$, $N = 2$. Date-1 losses noise traders $\equiv E(\pi_{NT,1})$, date-2 losses noise traders $\equiv E(\pi_{NT,2})$, date-1 price efficiency $\equiv Var(v|p_1)$, date-2 price efficiency $\equiv Var(v|p_2, p_1)$, date-1 market depth $\equiv 1/\lambda_1$, date-2 market depth $\equiv 1/\lambda_2$.

We obtain a unique equilibrium, i.e., there exists a unique pair $(\lambda_1, \lambda_2) \in \mathbb{R}_{++}^2$ solving the system in (12) and (13).⁹ Figure 1 displays the results of the numerical simulation. Strikingly, expected noise trader losses are increasing in c at both trading dates. Analogously to the static model in Section 2, social trading opens up new profit opportunities for rational speculators on the back of retail traders. Retail investors would be better off if they did not engage in any form of social trading (i.e., $c = 0$). Interestingly, price efficiency at the early and the late trading dates now depends positively on the parameter c . Date-1 and date-2 market depths also profit for sufficiently small values of comovement. That is, while retail investors unambiguously suffer from coordinating their trading behavior, market quality can benefit through higher price efficiency and higher market liquidity. Furthermore, rational speculators raise their date-1 and date-2 trading intensities against noise as c rises. Notably, date-1 trading intensities are lower than their date-2 counterparts. At date 1, rational speculators take their date-2 trading behavior against date-1 noise trading into account, which makes them trade less aggressively against noise.

Empirical approximation of retail investors' expected losses

Our analyses of the static and dynamic models uncover that retail investors unequivocally suffer from engaging in social trading. In the following, we would like to approximate empirically how strongly retail investors suffer from trading in a correlated manner. Haghighi et al. (2024) provide an empirical proxy for correlation between retail investors' trades by resorting to data from the US online broker Robinhood between May 2018 and August 2020. We restrict our attention to the time span between March 2020 and August 2020, as retail investing has experienced a significant boost since the COVID-19 pandemic. For this time period, the authors compute an average correlation of 0.106. Transferring this value to our model, we set $c = 0.106$ (given $\sigma_u^2 = 1$, $\sigma_v^2 = 0.00036$, and $N = 2$) and compare retail investors' ensuing date-1 and date-2 expected losses to the case without social trading (i.e., $c = 0$). Direct computations reveal that, with social trading, retail investors' expected losses are 5.1% higher than without social trading. Thus, based on our model, for each US dollar invested, retail investors lose more than \$0.05 on average through social trading.¹⁰

⁹We conducted an extensive set of numerical simulations. In all cases, equilibrium is unique.

¹⁰For the whole time span between May 2018 and August 2020, Haghighi et al. (2024) obtain an average correlation of 0.076, which amounts to a loss of 4% due to social trading.

4 Extensions and Variations

The models of Sections 2 and 3 derive our new insights within the simplest framework possible. In this section, we extend the baseline framework by allowing rational speculators to observe more than one noise component simultaneously and by adding intertemporal correlation among noise trading, respectively. The ensuing analyses reveal that our main results remain robust: retail investors still suffer from engaging in social trading and market quality tends to benefit.

4.1 Observing multiple noise components

Static model

The extended static model is the same as in Section 2 except for one difference: Instead of observing one noise component only, each rational speculator now observes $k \geq 1$ out of $N \geq 2$ noise components. We assume that k divides N as an integer. This means that there are N/k rational speculators in total. Speculator i 's demand function is given by $d_i = \alpha_i v - \beta_i \sum_{j=1}^k u_{ij}$, for $i = 1, 2, \dots, N/k$. Solving for equilibrium analogously to Section 2 yields (with the proof delegated to Appendix A):

Proposition 6: *There exists a unique linear equilibrium with N/k rational speculators, in which*

$$\begin{aligned}\alpha_i &= \frac{k}{(N+k)\lambda}, \\ \beta_i &= \frac{(N-1)c + \sigma_u^2}{(N+k-2)c + 2\sigma_u^2}, \quad \text{for } i = 1, 2, \dots, N/k, \\ \lambda &= \frac{\sqrt{k}[c(N+k-2) + 2\sigma_u^2]\sigma_v}{(N+k)[c(k-1) + \sigma_u^2]\sqrt{c(N-1) + \sigma_u^2}}.\end{aligned}$$

From Proposition 6, we see that rational speculators' trading intensity against the observed noise components is negatively correlated to the number of noise components each rational speculator observes (i.e., $\partial\beta_i/\partial k < 0$). If the market is more concentrated, i.e., there are fewer noise components that speculator i does not observe, she responds by scaling down her trading intensity against noise.

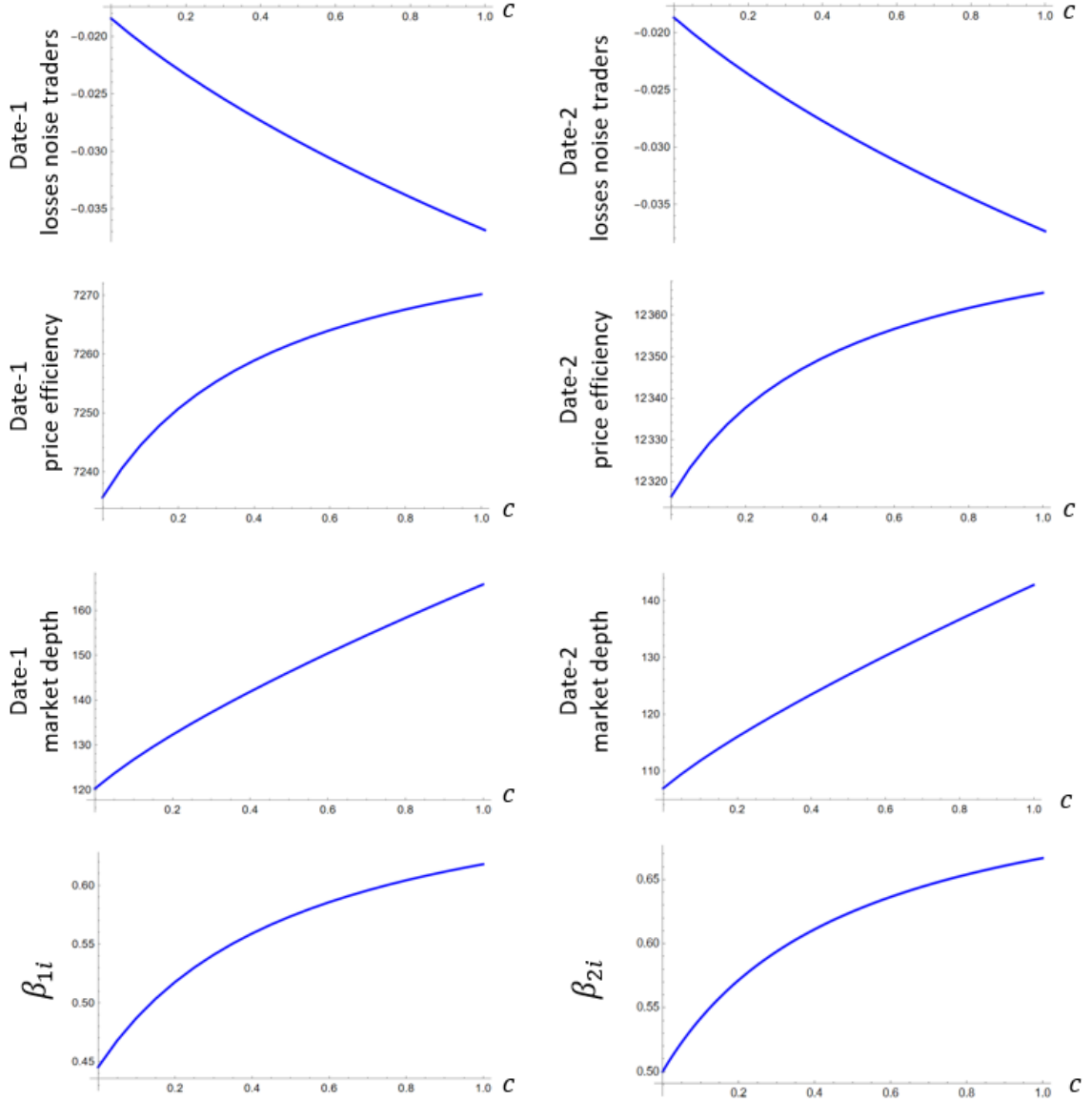


Figure 2: **Comparative statics in the dynamic model (observing multiple noise components)**. Parameters: $\sigma_v^2 = 0.00036$, $\sigma_u^2 = 1$, $N = 4$, $k = 2$. Date-1 losses noise traders $\equiv E(\pi_{NT,1})$, date-2 losses noise traders $\equiv E(\pi_{NT,2})$, date-1 price efficiency $\equiv Var(v|p_1)$, date-2 price efficiency $\equiv Var(v|p_2, p_1)$, date-1 market depth $\equiv 1/\lambda_1$, date-2 market depth $\equiv 1/\lambda_2$.

Market Properties

The effect of social trading on market liquidity and retail investors' expected wealth in the extended framework is as follows:

Proposition 7: *In equilibrium,*

- (a) *for k large enough, market depth depends positively on social trading: $\partial(1/\lambda)/\partial c > 0$,*
- (b) *noise traders' expected losses are increasing in c : $\partial E(\pi_{NT})/\partial c < 0$.*

The proof can be found in Appendix A. In the extension with N/k rational speculators, retail investors still unequivocally suffer from engaging in social trading. Their expected wealth remains a decreasing function of the intensity of social trading c , regardless of the value of k . Analyzing the effect on market liquidity, however, turns out to be more complicated than in the baseline model of Section 2. For k being sufficiently small, market liquidity can depend in a non-monotonic way on c . In this case, market depth benefits from social trading for small and large values of c and suffers from it for intermediate values of c (see Appendix A for more details). If k is sufficiently large, by contrast, market liquidity unambiguously profits from social trading. In this scenario, social trading attenuates the market makers' adverse selection problem. Recall that rational speculators offset relatively little of noise traders' influence (i.e., β_i is small) if k is large. As a consequence, the alleviating impact on adverse selection stemming from social trading injecting an additional source of noise into aggregate order flow dominates the intensifying impact of speculators counteracting the influence of noise trading. This attenuates the adverse selection problem, inducing market makers to set a price that is less sensitive to changes in aggregate order flow. This raises market liquidity.

Dynamic model

Analogously to the static setup, each rational speculator now observes k out of N noise components at each of the two trading dates. The rest of the assumptions of Section 3 stays unchanged. Solving the model backwards yields:

Proposition 8: *In a linear dynamic equilibrium with N/k speculators, it holds that*

$$\alpha_{2i} = \frac{k}{(N+k)\lambda_2},$$

$$\beta_{2i} = \frac{(N-1)c + \sigma_u^2}{(N+k-2)c + 2\sigma_u^2},$$

$$\begin{aligned}
e_{2i} &= \frac{k}{(N+k)\lambda_2}, \\
\alpha_{1i} &= \frac{k[-k^2(2\lambda_1 - \lambda_2) + 2k\lambda_2N + \lambda_2N^2]}{\lambda_1(k^3\lambda_2 + 3k\lambda_2N^2 + \lambda_2N^3 - 2k^2\lambda_1N + 3k^2\lambda_2N)} \\
\beta_{1i} &= \frac{[-k^2(2\lambda_1 - \lambda_2) + 2k\lambda_2N + \lambda_2N^2][c(N-1) + \sigma_u^2]}{c[k^3\lambda_2 + \lambda_2N^2(N-2) + k\lambda_2N(3N-4) + k^2(2\lambda_1 - 2\lambda_2 - 2\lambda_1N + 3\lambda_2N)] + 2\sigma_u^2[k^2(\lambda_2 - \lambda_1) + 2k\lambda_2N + \lambda_2N^2]},
\end{aligned}$$

for $i = 1, 2, \dots, N/k$. The values of λ_1 and λ_2 are obtained by solving

$$\begin{aligned}
\lambda_1 &= \frac{(N/k)\alpha_{1i}\sigma_v^2}{[(N/k)\alpha_{1i}]^2\sigma_v^2 + N(1 - \beta_{1i})^2\sigma_u^2 + (N-1)N(1 - \beta_{1i})^2c}, \\
\lambda_2 &= \frac{[(N/k)\alpha_{1i}Cov(w_1, w_2) - (N/k)\bar{\alpha}_{2i}Var(w_1)]\sigma_v^2}{Cov^2(w_1, w_2) - Var(w_1)Var(w_2)}.
\end{aligned}$$

The proof as well as the expressions for $\bar{\alpha}_{2i}$, $Cov(w_1, w_2)$, $Var(w_1)$, and $Var(w_2)$ can be found in Appendix A. As in Section 3, the overall complexity of the dynamic model precludes a full analytical characterization of equilibrium. Instead, we simulate the model numerically for a given set of parameter values. Figure 2 depicts the impact of social trading on market quality and noise traders' expected losses. As in the baseline model of Section 3, noise traders clearly suffer from social trading as their expected wealth remains a monotonically decreasing function of the parameter c . Similarly to the baseline model, market quality benefits through higher price efficiency at both trading dates. Remarkably, date-1 and date-2 market depths are now increasing in c , rather than hump-shaped as in the model of Section 3. That is, in the dynamic extension with speculators observing multiple noise signals, market quality tends to benefit even more through social trading than in the baseline model with observing one noise component only.

4.2 Intertemporal noise trader correlation

In this subsection, we extend the baseline dynamic framework of Section 3 by adding correlation between noise trading across the two trading dates. More specifically, we assume $Cov(u_{1i}, u_{2i'}) = \gamma$ ($i, i' = 1, 2, \dots, N$), with $0 < \gamma \leq \sigma_u^2$. The rest of the assumptions remains unchanged (i.e., there is still within-period correlation at both trading dates equal to c). Introducing intertemporal correlation adds additional complexity to the model, preventing

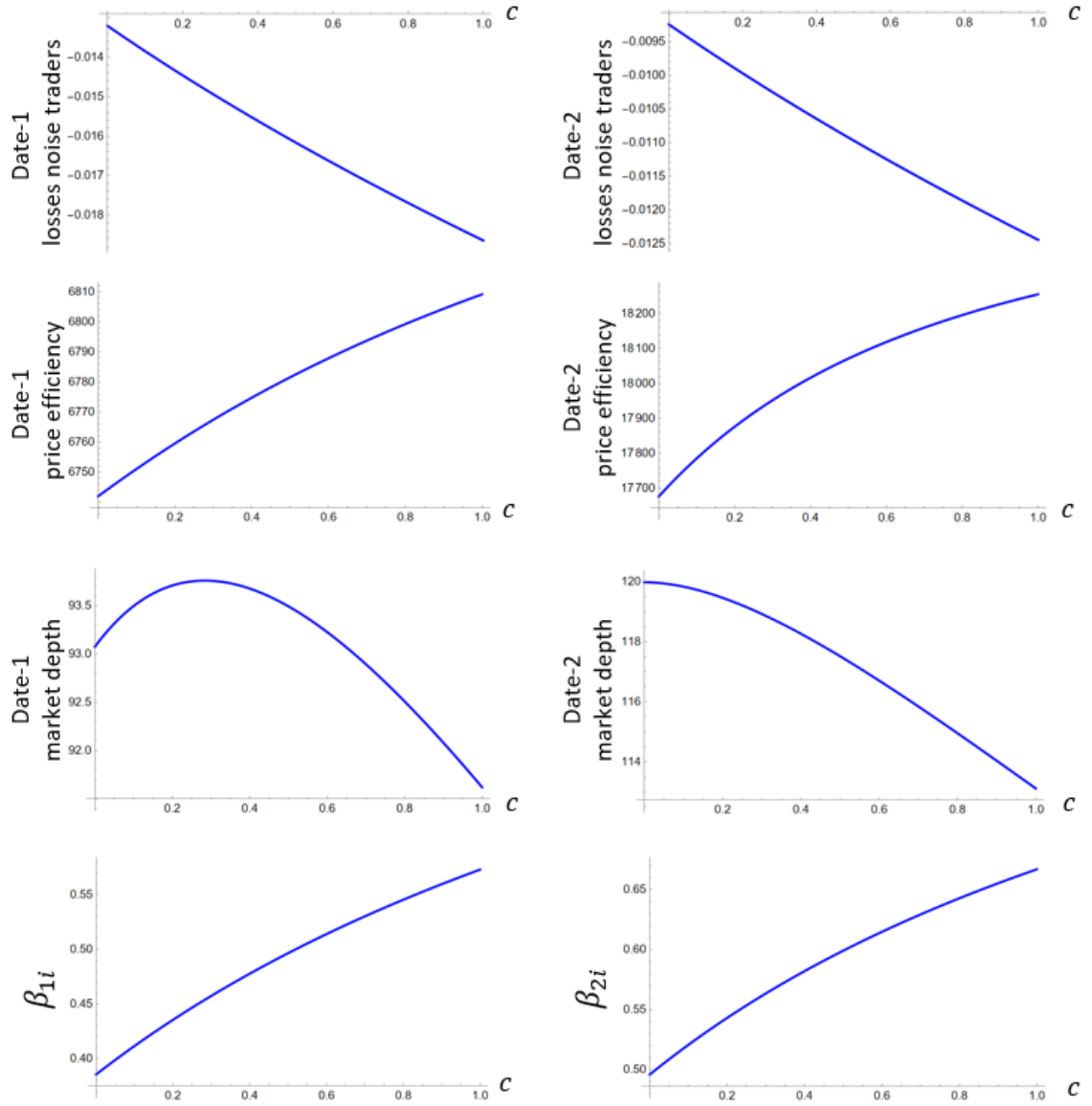


Figure 3: **Comparative statics in the dynamic model (intertemporal correlation).** Parameters: $\sigma_v^2 = 0.00036$, $\sigma_u^2 = 1$, $N = 2$, $\gamma = 0.106$. Date-1 losses noise traders $\equiv E(\pi_{NT,1})$, date-2 losses noise traders $\equiv E(\pi_{NT,2})$, date-1 price efficiency $\equiv Var(v | p_1)$, date-2 price efficiency $\equiv Var(v | p_2, p_1)$, date-1 market depth $\equiv 1/\lambda_1$, date-2 market depth $\equiv 1/\lambda_2$.

an analytical characterization of equilibrium. Appendix B provides details on how to determine the dynamic linear equilibrium. To check whether our obtained results are robust to intertemporal correlation, we set $\gamma = 0.106$ (given $\sigma_u^2 = 1$), in line with the empirical proxy of Haghighi et al. (2024), and conduct a comparative-statics analysis with respect to the parameter c . Figure 3 depicts the ensuing impact on noise traders’ expected wealth and on the market quality measures. Noise traders still unequivocally suffer from engaging in social trading. As in the baseline model of Section 3, date-1 and date-2 price efficiencies are monotonically increasing functions of c . Date-1 market liquidity is still hump-shaped in c . Date-2 market liquidity, by contrast, is now monotonically decreasing in c , rather than hump-shaped as in the baseline model. That is, with intertemporal noise trader correlation, market quality still tends to benefit from social trading, albeit to a lesser extent than without intertemporal correlation.

5 Conclusion

Social media is significantly influencing the behavior of both modern professional and retail investors in financial markets. Retail traders increasingly rely on advice and information gathered from various social media sources, including Finfluencers, stock message boards, and other popular online platforms, to devise their investment strategies. This phenomenon, known as social trading, has led to stronger coordination and herding behavior among retail investors. Striking examples of social trading include the recent meme stock episodes involving GameStop, AMC Entertainment, and others.

Professional investors, such as hedge funds, are aware of these new developments and have resorted to advances in big data analysis and artificial intelligence to gather information about the trading behavior of retail investors. Using a theoretical model à la Kyle (1985) with non-fundamental speculation, we explore the consequences of social trading by introducing comovement into retail traders’ market orders. Our findings reveal that social trading *harms* rather than benefits retail investors. The herding behavior induced by social trading creates new profit opportunities for professional investors at the expense of retail traders.

Remarkably, the effects on market quality tend to be positive. Both price efficiency and market liquidity tend to benefit from social trading. While social trading can thus have positive consequences for the efficiency and well-functioning of financial markets, it produces “collateral damage” in the form of worsening retail traders’ expected losses. Put differently,

from a policy perspective, fears that social trading could jeopardize price efficiency and financial stability can be dismissed based on our model results. However, in line with the warnings issued by relevant regulatory bodies such as the SEC and Germany's BaFin, our results should serve as a cautionary signal for retail investors. They should consider the financial risks linked to trading based on advice and information obtained through social media.

References

- Arnold, Lutz G., and David Russ (2024), “Listening to the noise: On price efficiency with dynamic trading”, *International Review of Economics and Finance* 93, 103–120.
- Arrowood, Spencer B. (2024), “TikTok: Is it Time to Regulate ”Finfluencer” Investment Advice on Social Media?”, *N.C. Banking Inst.* 429.
- Bernhardt, Dan, and Bart Taub (2008), “Front-running dynamics”, *Journal of Economic Theory* 138, 288–296.
- Cheyne, Edwige, and Carolyn B. Levine (2012), “Analysts’ Sale and Distribution of Non-Fundamental Information”, *Review of Accounting Studies* 17, 352–388.
- Cespa, Giovanni, and Xavier Vives (2012), “Dynamic Trading and Asset Prices: Keynes vs. Hayek”, *Review of Economic Studies* 79, 539–580.
- Cespa, Giovanni, and Xavier Vives (2015), “The Beauty Contest and Short-Term Trading”, *Journal of Finance* 70, 2099–2154.
- Demarquette, Maximilien (2016), “Strategic Trading with Endogenous Fundamental and Non-Fundamental Information Acquisition”, *Working Paper*.
- Eaton, Gregory W., T. Clifton Green, Brian S. Roseman, and Yanbin Wu (2022), “Retail Trader Sophistication and Stock Market Quality: Evidence from Brokerage Outages”, *Journal of Financial Economics* 146, 502–528.
- Farboodi, Maryam, and Laura Veldkamp (2020), “Long-Run Growth of Financial Data Technology”, *American Economic Review* 110, 2485–2523.
- Foucault, Thierry, and Laurence Lescourret (2003), “Information Sharing, Liquidity and Transaction Costs”, *Finance* 24, 45–78.
- Ganguli, Jayant Vivek, and Liyan Yang (2009), “Complementarities, Multiplicity, and Supply Information”, *Journal of the European Economic Association* 7, 90–115.
- Grossman, Sanford J., and Joseph E. Stiglitz (1980), “On the Impossibility of Informationally Efficient Markets”, *American Economic Review* 70, 393–408.

- Guan, Sue S. (2023), “The Rise of the Finfluencer”, *Journal of Law & Business* 19, 489–563.
- Haghighi, Afshin, Robert Faff, and Barry Oliver (2024), “Retail traders and co-movement: Evidence from Robinhood trading activity”, *International Review of Financial Analysis* 95, 103431.
- Hull, Isaiah, and Yingjie Qi (2024), “The Impact of Finfluencers on Retail Investment”, *Working Paper*.
- Keasey, Kevin, Costas Lambrinoudakis, Danilo V. Mascia, and Zhengfa Zhang (2025), “The impact of social media influencers on the financial market performance of firms”, *European Financial Management* 31, 745–785.
- Kyle, Albert (1985), “Continuous Auctions and Insider Trading”, *Econometrica* 53, 1315–1335.
- Madrigal, Vincente (1996), “Non-Fundamental Speculation”, *Journal of Finance* 51, 553–578.
- Manzano, Carolina, and Xavier Vives (2011), “Public and Private Learning From Prices, Strategic Substitutability and Complementarity, and Equilibrium Multiplicity”, *Journal of Mathematical Economics* 47, 346–369.
- Marmora, Paul, and Oleg Rytchkov (2018), “Learning about Noise”, *Journal of Banking & Finance* 89, 209–224.
- Peress, Joel, and Daniel Schmidt (2019), “Glued to the TV: Distracted Noise Traders and Stock Market Liquidity”, *Journal of Finance* 75, 1083–1133.
- Peress, Joel, and Daniel Schmidt (2021), “Noise Traders Incarnate: Describing a Realistic Noise Trading Process”, *Journal of Financial Markets* 54, 100618.
- Rochet, Jean-Charles, and Jean-Luc Vila (1994), “Insider Trading without Normality”, *The Review of Economic Studies* 61, 131–152.
- Röell, Ailsa (1990), “Dual-capacity trading and the quality of the market”, *Journal of Financial Intermediation* 1, 105–124.

- Russ, David (2022), “Multidimensional noise and non-fundamental information diversity”, *The North American Journal of Economics and Finance* 59, 101593.
- Sadzik, Tomasz, and Chris Woolnough (2021), “Rational Destabilization in a Frictionless Market”, *Journal of Economic Theory* 192, 105169.
- Yang, Liyan, and Haoxiang Zhu (2017), “Non-Fundamental Speculation Revisited”, *Journal of Finance* 72, 2759–2772.
- Yang, Liyan, and Haoxiang Zhu (2020), “Back-Running: Seeking and Hiding Fundamental Information in Order Flows”, *The Review of Financial Studies* 33, 1484–1533.
- Waters, Brian, Xingtang Zhang, Wei Zhou, and Shrihari Santosh (2024), “Manipulating Non-Fundamental Information”, *SSRN Working Paper*.

A Proofs

Proof of Proposition 1: By (1) and (2), speculator i 's expected profit is equal to

$$\mathbb{E}[(v - p)d_i \mid v, u_i] = \left\{ v - \lambda \left[d_i + \sum_{i' \neq i}^N \alpha_{i'} v + \sum_{i' \neq i}^N (1 - \beta_{i'}) (c/\sigma_u^2) u_{i'} + u_i \right] \right\} d_i.$$

The FOC in d_i is thus given by

$$-2\lambda d_i + v - \lambda \left[\sum_{i' \neq i}^N \alpha_{i'} v + \sum_{i' \neq i}^N (1 - \beta_{i'}) (c/\sigma_u^2) u_{i'} + u_i \right] = 0.$$

Solving for d_i yields

$$d_i = \frac{1 - \lambda \sum_{i' \neq i}^N \alpha_{i'}}{2\lambda} v - \frac{1 + (c/\sigma_u^2) \sum_{i' \neq i}^N (1 - \beta_{i'})}{2} u_i. \quad (\text{A.1})$$

Matching coefficients with (2) delivers

$$\begin{aligned} \alpha_i &= \frac{1 - \lambda \sum_{i' \neq i}^N \alpha_{i'}}{2\lambda}, \\ \beta_i &= \frac{1 + (c/\sigma_u^2) \sum_{i' \neq i}^N (1 - \beta_{i'})}{2}. \end{aligned}$$

Taking into account the symmetry of rational speculators' trading intensities, we can easily arrive at the expressions for α_i and β_i in the proposition.

The market makers' signal extraction problem boils down to

$$\lambda = \frac{N\alpha_i\sigma_v^2}{(N\alpha_i)^2\sigma_v^2 + N(1 - \beta_i)^2\sigma_u^2 + (N - 1)N(1 - \beta_i)^2c}.$$

Substituting for the expressions for α_i and β_i and solving for λ yields the equation in the proposition. $\square$

Proof of Proposition 4: Case with $N - 1$ non-fundamental speculators and social trading:

By (A.1), if $\beta_N = 0$, we get

$$\beta_i = \frac{c(N - 1) + \sigma_u^2}{c(N - 2) + 2\sigma_u^2}.$$

Accordingly, the sensitivity of the market price to aggregate order flow is given by:

$$\lambda = \frac{N\alpha_i\sigma_v^2}{(N\alpha_i)^2\sigma_v^2 + (N-1)(1-\beta_i)^2\sigma_u^2 + \sigma_u^2 + [(N-2)(1-\beta_i) + 2](N-1)(1-\beta_i)c}.$$

Substituting for α_i and β_i and solving for λ yields

$$\lambda = \frac{\sqrt{N}\sigma_v[c(N-2) + 2\sigma_u^2]}{(N+1)\sqrt{-c^3(N^2-3N+2) + c^2(N^2-7N+7) + c(N^2+3N-8)\sigma_u^4 + \sigma_u^6(N+3)}}.$$

The cubic polynomial under the square root above, $f(c)$ say, exhibits one sign change for $N \geq 2$. By Descartes' rule of signs, it has a single positive real root. We can show that $f(0) > 0$ and $f(\sigma_u^2) > 0$. Thus, the cubic polynomial takes on positive values for $0 < c \leq \sigma_u^2$, which means $\lambda > 0$.

Consequently, noise traders' expected losses boil down to

$$\begin{aligned} E(\pi_{NT}) &= -(N-1)\lambda[(1-\beta_i)\sigma_u^2 + (N-2)(1-\beta_i)c + c] - \lambda[\sigma_u^2 + (N-1)(1-\beta_i)c] \\ &= -\frac{\sqrt{N}\sigma_v[-c^2(N-1) + c(N^2-2)\sigma_u^2 + (N+1)\sigma_u^4]}{(N+1)\sqrt{f(c)}}. \end{aligned} \quad (\text{A.2})$$

The numerator as well as the denominator in (A.2) are positive for $0 < c \leq \sigma_u^2$, which yields $E(\pi_{NT}) < 0$.

Case without non-fundamental speculation and social trading: For β_i ($i = 1, 2, \dots, N$) = $c = 0$, the signal-to-noise ratio becomes

$$\lambda = \frac{N\alpha_i\sigma_v^2}{(N\alpha_i)^2\sigma_v^2 + N\sigma_u^2}.$$

Substituting for α_i and rearranging yields

$$\lambda = \frac{\sigma_v}{(N+1)\sigma_u}.$$

Noise traders' expected losses are thus equal to

$$E(\pi_{NT}) = NE[(v-p)u_i] = -\frac{N}{N+1}\sigma_v\sigma_u < 0. \quad (\text{A.3})$$

By (A.2) and (A.3), the condition in the proposition requires

$$-\frac{N}{N+1}\sigma_v\sigma_u > g(c) \equiv -\frac{\sqrt{N}\sigma_v[-c^2(N-1) + c(N^2-2)\sigma_u^2 + (N+1)\sigma_u^4]}{(N+1)\sqrt{f(c)}}. \quad (\text{A.4})$$

Note that, for $c = 0$, the RHS in (A.4) boils down to

$$g(0) = -\sqrt{\frac{N}{N+3}}\sigma_v\sigma_u, \quad (\text{A.5})$$

which is greater than the LHS in (A.4) for $N \geq 2$.

For $c = \sigma_u^2$, the RHS in (A.4) becomes

$$g(\sigma_u^2) = -\frac{\sqrt{N^3}}{(N+1)}\sigma_v\sigma_u, \quad (\text{A.6})$$

which is smaller than the LHS in (A.4) for $N \geq 2$.

Furthermore, straightforward but rather tedious calculations reveal:

$$\frac{\partial g(c)}{\partial c} = -\frac{(N-1)\sqrt{N}\sigma_v h(c)}{2i(c)(N+1)\sqrt{f(c)}},$$

where

$$\begin{aligned} h(c) &\equiv (N-2)c^3 + (N^2-3N+8)c^2 - \sigma_u^4(10+N)c + \sigma_u^6(N^2+3N+4), \\ i(c) &\equiv -(N-2)c^2 + \sigma_u^2(N-5)c + \sigma_u^4(N+3). \end{aligned}$$

Note that $h(c)$ exhibits one sign change and therefore one positive real root for $N \geq 2$. Furthermore, $h(0) > 0$ and $h(\sigma_u^2) > 0$, which means that $h(c) > 0$ for $0 < c \leq \sigma_u^2$. For $N = 2$, the polynomial $i(c)$ exhibits one sign change. Together with $i(0) > 0$ and $i(\sigma_u^2) > 0$, $i(c)$ is positive for $0 < c \leq \sigma_u^2$. For $N > 2$, $i(c)$ still exhibits one sign change and we still have $i(0) > 0$ and $i(\sigma_u^2) > 0$. Thus, $i(c) > 0$ for $0 < c \leq \sigma_u^2$. Putting all results together yields:

$$\frac{\partial g(c)}{\partial c} = -\frac{(N-1)\sqrt{N}\sigma_v h(c)}{2i(c)(N+1)\sqrt{f(c)}} < 0,$$

for $0 < c \leq \sigma_u^2$. Thus, with $N-1$ non-fundamental speculators and social trading, noise traders' expected wealth is decreasing in c . Together with (A.5) and (A.6), this proves that

there exists a unique value of $c < \sigma_u^2$, $\bar{c}$ say, above which the condition in (A.4) is satisfied. $\square$

Proof of Proposition 5: Speculator i chooses her market orders d_{1i} and d_{2i} to maximize expected lifetime profits:

$$\mathbb{E}\{(v - p_1)d_{1i} + \mathbb{E}[(v - p_2)d_{2i} \mid v, u_{1i}, u_{2i}, p_1] \mid v, u_{1i}\}.$$

We solve the problem backwards, i.e., we first determine d_{2i} . By (8), (9), and (11), the innermost expected value operator is equal to

$$\begin{aligned} & \mathbb{E}[(v - p_2)d_{2i} \mid v, u_{1i}, u_{2i}, p_1] \\ &= \left\{ v - p_1 - \lambda_2 \left[d_{2i} + \sum_{i' \neq i}^N \alpha_{2i'} v + \sum_{i' \neq i}^N (1 - \beta_{2i'}) (c/\sigma_u^2) u_{2i'} - \sum_{i' \neq i}^N e_{2i'} p_1 + u_{2i} \right] \right\} d_{2i}. \end{aligned}$$

The FOC in d_{2i} is

$$-2\lambda_2 d_{2i} + v - p_1 - \lambda_2 \left[\sum_{i' \neq i}^N \alpha_{2i'} v + \sum_{i' \neq i}^N (1 - \beta_{2i'}) (c/\sigma_u^2) u_{2i'} - \sum_{i' \neq i}^N e_{2i'} p_1 + u_{2i} \right] = 0.$$

Solving for d_{2i} yields

$$d_{2i} = \frac{1 - \lambda_2 \sum_{i' \neq i}^N \alpha_{2i'}}{2\lambda_2} v - \frac{1 + \sum_{i' \neq i}^N (1 - \beta_{2i'}) (c/\sigma_u^2)}{2} u_{2i} - \frac{1 - \lambda_2 \sum_{i' \neq i}^N e_{2i'}}{2\lambda_2} p_1. \quad (\text{A.7})$$

Matching coefficients with (11) delivers

$$\begin{aligned} \alpha_{2i} &= \frac{1 - \lambda_2 \sum_{i' \neq i}^N \alpha_{2i'}}{2\lambda_2}, \\ \beta_{2i} &= \frac{1 + \sum_{i' \neq i}^N (1 - \beta_{2i'}) (c/\sigma_u^2)}{2}, \\ e_{2i} &= \frac{1 - \lambda_2 \sum_{i' \neq i}^N e_{2i'}}{2\lambda_2}. \end{aligned}$$

Recalling the symmetry of trading strategies, we can easily solve for the final expressions of α_{2i} , β_{2i} , and e_{2i} .

Substituting (A.7) back into the expression for speculator i 's expected lifetime profits delivers

$$\begin{aligned} & \mathbb{E}\{(v - p_1)d_{1i} + \mathbb{E}[(v - p_2)d_{2i} \mid v, u_{1i}, u_{2i}, p_1] \mid v, u_{1i}\} \\ &= \mathbb{E} \left((v - p_1)d_{1i} + \frac{\left\{ [1 - \lambda_2 e_{2i}(N - 1)]\sigma_u^2 p_1 + [(1 - \beta_{2i})\lambda_2 c(N - 1) + \lambda_2 \sigma_u^2] u_{2i} \right\}^2}{4\lambda_2 \sigma_u^4} \mid v, u_{1i} \right). \end{aligned}$$

Recalling (8), (10), and $a_{2i} = e_{2i} = 1/[(N + 1)\lambda_2]$, the FOC in d_{1i} is

$$\begin{aligned} & -2\lambda_1 d_{1i} + v - \lambda_1[(N - 1)\alpha_{1i}v - (N - 1)\beta_{1i}(c/\sigma_u^2)u_{1i} + u_{1i} + (N - 1)(c/\sigma_u^2)u_{1i}] \\ & + \frac{2\lambda_1^2}{(N + 1)^2\lambda_2}d_{1i} + \frac{2\lambda_1^2[(N - 1)(1 - \beta_{1i})c + \sigma_u^2]}{(N + 1)^2\lambda_2\sigma_u^2}u_{1i} + \frac{[(N - 1)\alpha_{1i}\lambda_1 - 1]2\lambda_1}{(N + 1)^2\lambda_2}v = 0. \end{aligned}$$

Solving for d_{1i} gives

$$\begin{aligned} d_{1i} &= \frac{[2\lambda_1 - \lambda_2(N + 1)^2][1 - \lambda_1(N - 1)\alpha_i]}{2\lambda_1(\lambda_1 - (N + 1)^2\lambda_2)}v \\ & - \frac{[2\lambda_1 - (N + 1)^2\lambda_2][(N - 1)(1 - \beta_{1i})c + \sigma_u^2]}{2[\lambda_1 - (N + 1)^2\lambda_2]\sigma_u^2}u_{1i}. \end{aligned}$$

By (10),

$$\begin{aligned} \alpha_{1i} &= \frac{[2\lambda_1 - \lambda_2(N + 1)^2][1 - \lambda_1(N - 1)\alpha_i]}{2\lambda_1(\lambda_1 - (N + 1)^2\lambda_2)}, \\ \beta_{1i} &= \frac{[2\lambda_1 - (N + 1)^2\lambda_2][(N - 1)(1 - \beta_{1i})c + \sigma_u^2]}{2[\lambda_1 - (N + 1)^2\lambda_2]\sigma_u^2}. \end{aligned}$$

Rearranging the above equations yields the solutions for α_{1i} and β_{1i} .

Market makers' date-1 signal extraction problem is

$$\lambda_1 = \frac{Cov(v, w_1)}{Var(w_1)} = \frac{N\alpha_{1i}\sigma_v^2}{(N\alpha_{1i})^2\sigma_v^2 + N(1 - \beta_{1i})^2\sigma_u^2 + N(N - 1)(1 - \beta_{1i})^2c}.$$

At date 2, we obtain

$$\lambda_2 = \frac{Cov(v, w_2 \mid w_1)}{Var(w_2 \mid w_1)}.$$

By the law of total covariance,

$$\text{Cov}(v, w_2 | w_1) = \text{Cov}(v, w_2) - \text{Cov}[\mathbb{E}(v | w_1), \mathbb{E}(w_2 | w_1)].$$

Further computations yield

$$\begin{aligned} \text{Cov}[\mathbb{E}(v | w_1), \mathbb{E}(w_2 | w_1)] &= \frac{N\alpha_{1i}\sigma_v^2\text{Cov}(w_1, w_2)}{\text{Var}(w_1)}, \\ \text{Cov}(v, w_2) &= N\bar{\alpha}_{2i}\sigma_v^2, \end{aligned}$$

where $\bar{\alpha}_{2i} \equiv \alpha_{2i} - e_{2i}\lambda_1 N\alpha_{1i}$. Moreover,

$$\text{Var}(w_2 | w_1) = \text{Var}(w_2) - \frac{\text{Cov}^2(w_1, w_2)}{\text{Var}(w_1)}.$$

Putting the obtain results together and simplifying delivers

$$\lambda_2 = \frac{[N\alpha_{1i}\text{Cov}(w_1, w_2) - N\bar{\alpha}_{2i}\text{Var}(w_1)]\sigma_v^2}{\text{Cov}^2(w_1, w_2) - \text{Var}(w_1)\text{Var}(w_2)},$$

where

$$\begin{aligned} \text{Var}(w_1) &= (N\alpha_{1i})^2 \sigma_v^2 + N(1 - \beta_{1i})^2 \sigma_u^2 + N(N - 1)(1 - \beta_{1i})^2 c, \\ \text{Var}(w_2) &= (N\bar{\alpha}_{2i})^2 \sigma_v^2 + N(1 - \beta_{2i})^2 \sigma_u^2 + N(N - 1)(1 - \beta_{2i})^2 c \\ &\quad + N^3 e_{2i}^2 \lambda_1^2 (1 - \beta_{1i})^2 [\sigma_u^2 + c(N - 1)], \\ \text{Cov}(w_1, w_2) &= N^2 \bar{\alpha}_{2i} \alpha_{1i} \sigma_v^2 - N^2 e_{2i} \lambda_1 (1 - \beta_{1i})^2 [\sigma_u^2 + c(N - 1)]. \end{aligned} \quad \square$$

Proof of Proposition 6: With observing k out of N noise trader components, speculator i 's expected profit is equal to

$$\begin{aligned} &\mathbb{E} \left[(v - p)d_i \mid v, \sum_{j=1}^k u_{ij} \right] \\ &= \left\{ v - \lambda \left[d_i + \sum_{i' \neq i}^{N/k} \alpha_{i'} v + \sum_{i' \neq i}^N (1 - \beta_{i'}) \mathbb{E} \left(u_{i'} \mid \sum_{j=1}^k u_{ij} \right) + \sum_{j=1}^k u_{ij} \right] \right\} d_i \\ &= \left\{ v - \lambda \left[d_i + (N/k - 1)\alpha_i v + (N - k)(1 - \beta_i) \frac{c}{c(k - 1) + \sigma_u^2} \sum_{j=1}^k u_{ij} + \sum_{j=1}^k u_{ij} \right] \right\} d_i. \end{aligned}$$

The FOC in d_i is

$$-2\lambda d_i + v - \lambda \left[(N/k - 1)\alpha_i v + (N - k)(1 - \beta_i) \frac{c}{c(k - 1) + \sigma_u^2} \sum_{j=1}^k u_{ij} + \sum_{j=1}^k u_{ij} \right] = 0.$$

Solving for d_i yields

$$d_i = \frac{1 - \lambda(N/k - 1)\alpha_i}{2\lambda} v - \frac{1 + \frac{(N - k)(1 - \beta_i)c}{c(k - 1) + \sigma_u^2}}{2} \sum_{j=1}^k u_{ij}.$$

Matching coefficients delivers

$$\alpha_i = \frac{1 - \lambda(N/k - 1)\alpha_i}{2\lambda},$$

$$\beta_i = \frac{1 + \frac{(N - k)(1 - \beta_i)c}{c(k - 1) + \sigma_u^2}}{2}.$$

By rearranging the two above equations, we can easily arrive at the expressions for α_i and β_i in the proposition.

The market makers' signal extraction problem boils down to

$$\lambda = \frac{(N/k)\alpha_i\sigma_v^2}{[(N/k)\alpha_i]^2\sigma_v^2 + N(1 - \beta_i)^2\sigma_u^2 + (N - 1)N(1 - \beta_i)^2c}.$$

Substituting for the expressions for α_i and β_i and solving for λ yields the equation in the proposition. $\square$

Proof of Proposition 7: To prove part (a), we differentiate λ from Proposition 6 with respect to c :

$$\frac{\partial \lambda}{\partial c} = - \frac{\sqrt{k}\sigma_v[(N - 1)(k - 1)(N + k - 2)c^2 - (N - 1)\sigma_u^2(N + 4 - 5k)c + 2(k - 1)\sigma_u^4]}{2(N + k)[c(k - 1) + \sigma_u^2]^2[c(N - 1) + \sigma_u^2]^{3/2}}.$$

The sign of the derivative depends on the sign of the term in square brackets in the numerator, which is a quadratic in c . Note that the coefficient of c^2 as well as the constant are always positive. Thus, the sign of the derivative depends crucially on the coefficient of c , whose sign is pinned down by the term $N + 4 - 5k$.

If $k \geq \frac{N+4}{5}$, the quadratic exhibits no sign change, and the derivative is negative for $0 < c \leq$

σ_u^2 . Whenever $1 < k < \frac{N+4}{5}$, the coefficient of c is negative, which means that the quadratic function exhibits two sign changes. Thus, according to Descartes' rule of signs, the quadratic has either two or zero positive real roots. The two positive real roots, c^* and c^{**} say, exist if the discriminant of the quadratic is non-negative. Thus, given a non-negative discriminant, λ is falling in c for $c < c^*$. For $c^* < c < c^{**}$, λ increases with c , and for $c > c^{**}$, λ is again falling in c . If the discriminant is negative, by contrast, λ is falling in c for $0 < c \leq \sigma_u^2$.

To prove part (b), recall that

$$E(\pi_{NT}) = -NCov(p, u_i) = -N\lambda[(1 - \beta_i)\sigma_u^2 + (N - 1)(1 - \beta_i)c].$$

Direct computations yield:

$$\frac{\partial E(\pi_{NT})}{\partial c} = -\frac{\sqrt{k}\sigma_v N[(k - 1)c + \sigma_u^2][(N - 1)c + \sigma_u^2]}{(N + k)[(k - 1)c + \sigma_u^2]\sqrt{(N - 1)c + \sigma_u^2}} < 0.$$

Thus, noise traders' expected wealth is decreasing in c . □

Proof of Proposition 8: Speculator i chooses her market orders d_{1i} and d_{2i} to maximize expected lifetime profits:

$$E\left\{(v - p_1)d_{1i} + E\left[(v - p_2)d_{2i} \mid v, \sum_{j=1}^k u_{1ij}, \sum_{j=1}^k u_{2ij}, p_1\right] \mid v, \sum_{j=1}^k u_{1ij}\right\}.$$

We solve the problem backwards. Determining d_{2i} works analogously to determining d_i in the static framework. Applying the analogous steps as in the proof of Proposition 6 delivers

$$\begin{aligned}\alpha_{2i} &= \frac{k}{(N + k)\lambda_2}, \\ \beta_{2i} &= \frac{(N - 1)c + \sigma_u^2}{(N + k - 2)c + 2\sigma_u^2}, \\ e_{2i} &= \frac{k}{(N + k)\lambda_2}.\end{aligned}$$

Substituting d_{2i} back into the value function, determining the FOC in d_{1i} , and solving for

d_{1i} yields:

$$d_{1i} = - \frac{[k^2(2\lambda_1 - \lambda_2) - 2k\lambda_2N - \lambda_2N^2]\{c[\beta_{1i}(k - N) + N - 1] + \sigma_u^2\}}{2[k^2(\lambda_1 - \lambda_2) - 2k\lambda_2N - \lambda_2N^2][c(k - 1) + \sigma_u^2]} \sum_{j=1}^k u_{1ij} \\ + \frac{(k + \alpha_{1i}k\lambda_1 - \alpha_{1i}\lambda_1N)[k^2(2\lambda_1 - \lambda_2) - 2k\lambda_2N - \lambda_2N^2]}{2k\lambda_1[k^2(\lambda_1 - \lambda_2) - 2k\lambda_2N - \lambda_2N^2]} v.$$

Matching coefficients and solving for α_{1i} and β_{1i} delivers the expressions in the proposition.

Market makers' date-1 signal extraction problem is

$$\lambda_1 = \frac{Cov(v, w_1)}{Var(w_1)} = \frac{(N/k)\alpha_{1i}\sigma_v^2}{[(N/k)\alpha_{1i}]^2\sigma_v^2 + N(1 - \beta_{1i})^2\sigma_u^2 + N(N - 1)(1 - \beta_{1i})^2c}.$$

At date 2, applying the analogous steps as in the proof of Proposition 5, we obtain

$$\lambda_2 = \frac{[(N/k)\alpha_{1i}Cov(w_1, w_2) - (N/k)\bar{\alpha}_{2i}Var(w_1)]\sigma_v^2}{Cov^2(w_1, w_2) - Var(w_1)Var(w_2)},$$

where

$$\begin{aligned} \bar{\alpha}_{2i} &\equiv \alpha_{2i} - e_{2i}\lambda_1(N/k)\alpha_{1i}, \\ Var(w_1) &= [(N/k)\alpha_{1i}]^2\sigma_v^2 + N(1 - \beta_{1i})^2\sigma_u^2 + N(N - 1)(1 - \beta_{1i})^2c, \\ Var(w_2) &= [(N/k)\bar{\alpha}_{2i}]^2\sigma_v^2 + N(1 - \beta_{2i})^2\sigma_u^2 + N(N - 1)(1 - \beta_{2i})^2c \\ &\quad + (N/k)^2e_{2i}^2\lambda_1^2(1 - \beta_{1i})^2[N\sigma_u^2 + cN(N - 1)], \\ Cov(w_1, w_2) &= (N/k)^2\bar{\alpha}_{2i}\alpha_{1i}\sigma_v^2 - (N/k)e_{2i}\lambda_1(1 - \beta_{1i})^2[N\sigma_u^2 + cN(N - 1)]. \end{aligned} \quad \square$$

B Extension with intertemporal correlation

With intertemporal noise trader correlation, the derivation of equilibrium follows the same steps as in the model without across-period correlation. The degree of complexity, however, increases in such a way that displaying all steps linked to deriving equilibrium is not reasonable anymore. Instead, we provide a sketch of the proof in what follows. More details can be made available by the author upon request.

When there is correlation between noise trading across periods, date-1 noise trader demand explicitly influences a rational speculator's date-2 demand function. In general form, we

conjecture the following demand functions:

$$\begin{aligned}d_{1i} &= \alpha_{1i}v - \beta_{1i}u_{1i}, \\d_{2i} &= \alpha_{2i}v - \beta_{2i}u_{2i} - \delta_{2i}u_{1i} - e_{2i}p_1.\end{aligned}$$

As usual, speculator i chooses her market orders d_{1i} and d_{2i} to maximize expected lifetime profits:

$$\mathbb{E}\{(v - p_1)d_{1i} + \mathbb{E}[(v - p_2)d_{2i} \mid v, u_{1i}, u_{2i}, p_1] \mid v, u_{1i}\}.$$

We solve the problem backwards. The innermost expected value operator equals

$$\begin{aligned}&\mathbb{E}[(v - p_2)d_{2i} \mid v, u_{1i}, u_{2i}, p_1] \\&= \{v - p_1 - \lambda_2[d_{2i} + (N - 1)\alpha_{2i}v + (N - 1)(1 - \beta_{2i})\mathbb{E}(u_{2i'} \mid v, u_{1i}, u_{2i}, p_1) \\&\quad - (N - 1)\delta_{2i}\mathbb{E}(u_{1i'} \mid v, u_{1i}, u_{2i}, p_1) - (N - 1)e_{2i}p_1 + u_{2i}]\}d_{2i}.\end{aligned}$$

Note that speculator i now uses all of her available date-2 information to predict the date-2 and date-1 noise components observed by the other speculators.

After computing i 's date-2 expectations about the unobserved date-2 and date-1 noise components, we proceed to determine the FOC in d_{2i} . Afterwards, we solve for d_{2i} and match coefficients to determine the coefficient array $(\alpha_{2i}, \beta_{2i}, \delta_{2i}, e_{2i})$ as functions of $\alpha_{1i}, \beta_{1i}, \lambda_1, \lambda_2$ and the exogenous model parameters.

Putting the derived expression for d_{2i} back into the value function yields (for illustrative purposes, the solutions for $\alpha_{2i}, \beta_{2i}, \delta_{2i}$, and e_{2i} are not displayed):

$$\begin{aligned}&\mathbb{E}((v - p_1)d_{1i} + (\alpha_{2i}v - \beta_{2i}u_{2i} - \delta_{2i}u_{1i} - e_{2i}p_1) \\&\quad \times \{v - p_1 - \lambda_2[N\alpha_{2i}v + (N - 1)(1 - \beta_{2i})\mathbb{E}(u_{2i'} \mid v, u_{1i}, u_{2i}, p_1) \\&\quad - (N - 1)\delta_{2i}\mathbb{E}(u_{1i'} \mid v, u_{1i}, u_{2i}, p_1) - \delta_{2i}u_{1i} - Ne_{2i}p_1 + (1 - \beta_{2i})u_{2i}]\} \mid v, u_{1i}).\end{aligned}\quad (\text{B.1})$$

By the law of iterated expectations, we obtain

$$\begin{aligned}\mathbb{E}[\mathbb{E}(u_{2i'} \mid v, u_{1i}, u_{2i}, p_1) \mid v, u_{1i}] &= \mathbb{E}(u_{2i'} \mid v, u_{1i}) = (\gamma/\sigma_u^2)u_{1i}, \\ \mathbb{E}[\mathbb{E}(u_{1i'} \mid v, u_{1i}, u_{2i}, p_1) \mid v, u_{1i}] &= \mathbb{E}(u_{1i'} \mid v, u_{1i}) = (c/\sigma_u^2)u_{1i}.\end{aligned}$$

Furthermore, recall that i 's date-1 expectations about p_1 are equal to

$$E(p_1 | v, u_{1i}) = \lambda_1 [d_{1i} + (N-1)\alpha_{1i}v + (N-1)(1-\beta_{1i})(c/\sigma_u^2)u_{1i} + u_{1i}].$$

With all this in hand, we can compute the FOC of (B.1) in d_{1i} . Having derived the FOC, we solve for d_{1i} . We proceed to match coefficients to determine α_{1i} and β_{1i} as functions of λ_1, λ_2 and the exogenous model parameters.

Market makers' date-1 signal extraction problem is

$$\lambda_1 = \frac{Cov(v, w_1)}{Var(w_1)} = \frac{N\alpha_{1i}\sigma_v^2}{(N\alpha_{1i})^2\sigma_v^2 + N(1-\beta_{1i})^2\sigma_u^2 + N(N-1)(1-\beta_{1i})^2c}.$$

At date 2, we obtain

$$\lambda_2 = \frac{[N\alpha_{1i}Cov(w_1, w_2) - N\bar{\alpha}_{2i}Var(w_1)]\sigma_v^2}{Cov^2(w_1, w_2) - Var(w_1)Var(w_2)},$$

where

$$\begin{aligned} \bar{\alpha}_{2i} &\equiv \alpha_{2i} - e_{2i}\lambda_1 N\alpha_{1i}, \\ Var(w_1) &= (N\alpha_{1i})^2\sigma_v^2 + N(1-\beta_{1i})^2\sigma_u^2 + N(N-1)(1-\beta_{1i})^2c, \\ Var(w_2) &= (N\bar{\alpha}_{2i})^2\sigma_v^2 + N(1-\beta_{2i})^2[\sigma_u^2 + (N-1)c] + N\delta_{2i}^2[\sigma_u^2 + (N-1)c] + \\ &\quad N^3e_{2i}^2\lambda_1^2(1-\beta_{1i})^2[\sigma_u^2 + (N-1)c] + 2(1-\beta_{2i})\delta_{2i}N^2\gamma - \\ &\quad 2N^3e_{2i}\lambda_1(1-\beta_{1i})(1-\beta_{2i})\gamma + 2\delta_{2i}Ne_{2i}\lambda_1(1-\beta_{1i})[N\sigma_u^2 + N(N-1)c], \\ Cov(w_1, w_2) &= N^2\bar{\alpha}_{2i}\alpha_{1i}\sigma_v^2 + (1-\beta_{1i})(1-\beta_{2i})N^2\gamma - \\ &\quad \delta_{2i}(1-\beta_{1i})[N\sigma_u^2 + N(N-1)c] - Ne_{2i}\lambda_1(1-\beta_{1i})^2[N\sigma_u^2 + N(N-1)c]. \quad \square \end{aligned}$$