Technical Paper
A Growth-at-Risk model for the German economy

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Non-technical summary

Research Question

A key part of financial stability analysis includes quantifying the potential magnitude of downside risk to the economy related to financial vulnerabilities and financial market stress. However, analysing downside risk to the economy (i.e. ‘tail-risk’) is difficult, since the occurrence of severe financial stress in financial markets is relatively rare. While the appropriate measurement of tail-risk is clearly complicated, one empirical method that can be useful in this context is the Growth-at-Risk (GaR) methodology. The following paper describes the approach to evaluating a Growth-at-Risk model for quantifying the downside risk of Germany's overall economic output in relation to financial developments.

Contribution

This paper develops a systematic approach to evaluating several Growth-at-Risk models and describes the procedure for comparing forecasting capabilities at specific quantiles. As part of the evaluation, 200 model specifications are assessed based on the estimation of the 5% quantile and the 10% quantile. The evolution of tail risk to German GDP growth is discussed by employing skewed t-distributions for different points in time. The evaluation finds that the 10% quantile is more robust for analysing the downside risk to the economy than the 5% quantile. In addition, data for the COVID period is removed from the time series, since the COVID pandemic does not represent a crisis resulting from developments in the financial system.

Results

Our systematic comparison of model specification allows us to evaluate a model’s suitability to assess the effect on GDP growth, and we find the best model to include annualized GDP growth rates of the two previous periods, the Country-Level Index of Financial Stress for Germany (CLIFS), the National Financial Conditions Index of the USA (NFCI) and business confidence indicator (BCI) of German companies. The estimation results show that financial factors (CLIFS and NFCI) have reduced tail risks in recent years, while economic sentiment (BCI) has increased downside risks. At the same time, the tail risk for German GDP growth has generally declined over the past two years.

Nichttechnische Zusammenfassung

Fragestellung

Ein wichtiger Bestandteil der Finanzstabilitätsanalyse besteht darin, das potenzielle Ausmaß von Abwärtsrisiken für die Wirtschaft im Zusammenhang mit finanziellen Verwundbarkeiten und Finanzmarktstress zu quantifizieren. Die Analyse von Abwärtsrisiken für die Wirtschaft (d. h. „Tail-Risiken") ist jedoch schwierig, da das Auftreten deutlicher finanzieller Anspannungen auf den Finanzmärkten relativ selten ist. Obwohl die Messung des Tail-Risikos für die Wirtschaft kompliziert ist, kann die Growth-at-Risk (GaR)-Methodik hierbei hilfreich sein. Das vorliegende Papier beschreibt das Vorgehen zur Evaluation eines Growth-at-Risk-Modells für die Quantifizierung der Abwärtsrisiken der gesamtwirtschaftlichen Aktivität (BIP) Deutschlands in Abhängigkeit finanzieller Entwicklungen.

Beitrag

Dieses Papier entwickelt einen systematischen Ansatz zur Bewertung mehrerer Growth-at-Risk-Modelle und beschreibt das Verfahren zum Vergleich der Prognosefähigkeiten bei bestimmten Quantilen. Als Teil der Evaluation werden 200 Modellspezifikationen basierend auf der Schätzung des 5%-Quantils und des 10%-Quantils analysiert. GaR-Modelle für das Wachstum des deutschen BIP werden auf Grundlage des bewerteten Modells geschätzt. Die Entwicklung des Tail-Risikos für das Wachstum des deutschen BIP wird unter Verwendung von schiefen t-Verteilungen zu verschiedenen Zeitpunkten diskutiert. Die Evaluation zeigt, dass das 10%-Quantil für den GaR-Ansatz besser geeignet ist als das 5%-Quantil. Darüber hinaus werden Daten aus der COVID-Periode aus den Zeitreihen entfernt, da die COVID-Pandemie keine Krise darstellt, die aus Entwicklungen im Finanzsystem resultiert.

Ergebnisse

Durch einen systematischen Vergleich verschiedener Modellspezifikationen wird die Eignung eines Modells belegt, das die Entwicklung des annualisierten BIP-Wachstums durch das annualisierte BIP-Wachstum der zwei vorangegangenen Perioden, den Country-Level Index of Financial Stress für Deutschland (CLIFS), den National Financial Conditions Index der USA (NFCI) und einen Stimmungsindikator deutscher Unternehmen (BCI) berücksichtigt. Die Schätzergebnisse zeigen, dass finanzielle Faktoren (CLIFS und NFCI) in den letzten Jahren Tail-Risiken reduziert haben, während das wirtschaftliche Sentiment (BCI) die Abwärtsrisiken erhöht hat. Gleichzeitig ist das Tail-Risiko für das Wachstum des deutschen BIP in den vergangenen zwei Jahren insgesamt zurückgegangen.

A Growth-at-Risk model for the German economy*

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Abstract

This paper describes a Growth-at-Risk (GaR) model of the Bundesbank for Germany. This model takes the form of a quantile regression that quantifies downside risk to German GDP growth associated with financial developments. A systematic comparison of diverse model specifications is performed to select the most suitable GaR model based on economic criteria and out-of-sample predictive performance. The preferred model relates the 10% quantile of the conditional distribution of GDP growth to financial stress in Germany as captured by the Country-Level Index of Financial Stress (CLIFS), as well as US financial conditions as measured by the National Financial Conditions Index (NFCI) for the USA. In addition, the preferred specification includes GDP growth of the two preceding periods to account for serial dependence and a business confidence indicator (BCI) of German companies, which underscores that economic sentiment also matters for downside risk to growth. The evaluation shows that the 10% quantile coefficients are more stable than those of the 5% quantile, making the 10% quantile a more robust measure of downside risk for German GDP. Data from the COVID period are excluded, as the pandemic was not a financial system-driven crisis. Estimation results show that financial stress, measured by both CLIFS and NFCI, contributed most strongly to downside risk to GDP growth during the 2007/2008 Global Financial Crisis. The CLIFS also significantly increased downside risk in the early 2000s and following the Russian invasion of Ukraine. In recent years, historically low financial stress has corresponded to moderate downside risk, with economic sentiment acting as the main amplifier.

JEL classification: C53, E23, E27, E32, E44
Keywords: Growth-at-Risk, GDP Growth, Germany, Tail Risk, Financial Conditions

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1 Introduction

A key part of financial stability analysis and monitoring includes quantifying the potential magnitude of downside risk to the economy related to financial vulnerabilities and financial market stress.[2] In general, the relation between financial stress and severe downturns in the real economy is an important topic, at least since the 2007/2008 Global Financial Crisis (GFC). While vulnerabilities can build up in the financial system when the economy is expanding, including excessive leverage and asset price bubbles, a negative shock that hits such financial imbalances can result in a severe and longer recession or even a financial and banking crisis. Empirical evidence suggests that recessions accompanied by financial crises are typically much more severe than other recessions.[3] In general, macroprudential policy aims to provide resilience against such risk.[4] However, analysing downside risk to the economy (i.e. 'tail-risk') is difficult, since the occurrence of severe financial stress in financial markets is relatively rare. While the appropriate measurement of the tail-risk to the economy, and the potential impact of financial stress, is clearly complicated, one empirical method that can be useful in this context is the Growth-at-Risk (GaR) methodology.

This article presents the application of the Growth-at-Risk approach, first introduced by Adrian et al. (2019), to the German economy.[5] We therefore model the relationship between indicators of financial stress and the downside risk to the German economy. Downside risk in the real economy is measured using the lower end of the probability distribution for the quarterly growth rate of real gross domestic product (GDP). Based on quantile regressions, this methodology is employed to analyse how downside risk in the real economy fluctuates over time in connection with financial stress and financial vulnerabilities, while also allowing for the analysis of contributing factors. More specifically, this note describes the approach and results of a systematic evaluation of various Growth-at-Risk model specifications. The goal is to determine a model specification that adequately captures downside risks to German output growth as a function of financial developments. As part of the evaluation, 200 model specifications are assessed based on the estimation of the 5% quantile and the 10% quantile one quarter ahead, using data available up to the current quarter.

For the systematic evaluation, various combinations of variables from different categories are compared based on a predetermined metric. The categories include financial stress indicators, early-warning or financial-cycle indicators, variables capturing external factors and real economic indicators. In general, the evaluated model should yield plausible estimates for growth-

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[2] See e.g. Bundesbank (2021), Hafemann (2023), Herbst et al. (2024) and Prasad et al. (2019).

[3] See e.g. Prasad et al. (2019). Jordà et al. (2015) stress the importance of credit and leverage. See also Schularick and Taylor (2012).

[4] For more information on the application of GaR to financial stability analysis, see e.g. O'Brian and Wosser (2021).

[5] See also Adrian et al. (2022). See e.g. Hafemann (2023) for an application of the GaR framework to the German residential real estate market and for further information on the methodology.

at-risk one quarter ahead during periods of severe financial stress, and it should allow for monitoring downside risk under normal conditions. For this purpose, quantile scores are used as the main metric. The quantile score measures how well a prediction hits the desired quantile by weighting overestimates and underestimates differently. The quantile scores for the entire period, as well as an in-sample forecast for the period of the global financial crisis of 2007/08, are considered. This approach ensures that the model not only provides reliable forecasts during periods of heightened financial stress but also enables the monitoring of downside risks during more stable, normal conditions.

The findings suggest that employing the Country-Level Index of Financial Stress for Germany (CLIFS)[6], the National Financial Conditions Index of the USA (NFCI)[7], and the sentiment indicator of the manufacturing sector (BCI)[8], in addition to lagged values of German GDP as exogenous variables yield the optimal combination for estimating the 10% quantile. Furthermore, the evaluation revealed that the coefficients of the 10% quantile generally exhibit less variation than the coefficients of the 5% quantile. Therefore, the 10% quantile is more robust in the Growth-at-Risk analysis of the German economy. Since the COVID pandemic does not represent a crisis resulting from developments in the financial system, data for the COVID period is removed from the time series.[9] Using the evaluated model, the estimation results for 2025 Q2 GDP growth highlight the significant influence of financial factors. Low financial stress, measured by CLIFS and NFCI, reduces downside risks. Economic sentiment, reflected by the business confidence indicator, has increased downside risks to German GDP growth in recent years. The findings also show a general reduction in tail risk to GDP growth over the past two years.

2 Systematic evaluation of Growth-at-Risk models

To evaluate the predictive power of various indicators for downside risk to German GDP growth, we use a model comparison framework based on the Growth-at-Risk approach by Adrian et al. (2019). The primary objective is to determine which variables and model specifications best capture the lower tail of the conditional GDP growth distribution, representing periods of heightened downside risk. We use quantile regressions to assess the predictive power of variables across different financial and economic categories. We classify the variables into four groups: financial stress indicators, early warning or financial cycle indicators, external factors, and real economic indicators.

[6] See e.g. https://data.ecb.europa.eu/data/datasets/CLIFS/CLIFS.M.DE._Z.4F.EC.CLIFS_CI.IDX. For more information on the CLIFS and financial stress indicators in general, see Metiu (2022, 2024).
[7] See e.g. https://www.chicagofed.org/research/data/nfci/current-data.
[8] See e.g. https://www.oecd.org/en/data/indicators/business-confidence-index-bci.html.
[9] For precise definitions of crisis periods, see e.g. https://www.esrb.europa.eu/pub/financial-crises/html/index.en.html. The ESRB defines the COVID crisis as an ‘exogeneous health crisis due to Covid-19`.

The quantile regression framework estimates the relationship between future GDP growth y_{t+h} and independent variables x_t at various quantiles of the conditional distribution. We estimate the slope coefficient β_t for a specific quantile τ by minimizing the sum of quantile-weighted absolute error terms:

$$\hat{\beta}_\tau = \underset{\beta_\tau \in R}{\arg\min} \sum_{t=1}^{T-h} \left(\tau \cdot \mathbf{1}_{(y_{t+h} \geq x_t\beta)} |y_{t+h} - x_t\beta_\tau| + (1-\tau) \cdot \mathbf{1}_{(y_{t+h} < x_t\beta)} |y_{t+h} - x_t\beta_\tau|\right),$$

where $(\mathbf{1})$ is an indicator function, τ represents the quantile, and T is the end of the sample period. The predicted GDP growth at quantile τ, denoted as $\hat{y}_{t+h}$, is derived from the conditional quantile function:

$$\hat{Q}_{y_{t+h}|x_t}(\tau|x_t) = x_t\hat{\beta}_\tau$$

To reconstruct the conditional quantile function, we apply the skewed t-distribution introduced by Azzalini and Capitanio (2003). The probability density function of the skewed t-distribution is expressed as:

$$f(y; \mu, \sigma, \alpha, \nu) = \frac{2}{\sigma} t\left(\frac{y-\mu}{\sigma}; \nu\right) T\left(\alpha \frac{y-\mu}{\sigma} \sqrt{\frac{\nu+1}{\nu + \left(\frac{y-\mu}{\sigma}\right)^2}}; \nu+1\right)$$

where $t(\cdot;\nu)$ is the standard t-distribution with ν degrees of freedom and $T(\cdot;\nu+1)$ is the cumulative distribution function of the t-distribution with $\nu+1$ degrees of freedom. The skewed t-distribution is defined by four parameters: location (μ), scale (σ), skewness (α), and degrees of freedom (ν). This distribution accounts for asymmetry, enabling us to capture the skewness in the GDP growth distribution.

We estimate the parameters of the skewed t-distribution for each quarter t by minimizing the distance between the estimated quantile function $\hat{Q}_{y_{t+h}|x_t}(\tau|x_t)$ and the quantile function of the skewed t-distribution, $F^{-1}(\tau; \mu, \sigma, \alpha, \nu)$. We align the 5th, 25th, 75th, and 95th percentiles to ensure exact identification. Formally, the parameters are estimated as follows:

$$\{\hat{\mu}_{t+h}, \hat{\sigma}_{t+h}, \hat{\alpha}_{t+h}, \hat{\nu}_{t+h}\} = \underset{\mu,\sigma,\alpha,\nu}{\arg\min} \sum_{\tau} \left(\hat{Q}_{y_{t+h}|x_t}(\tau|x_t) - F^{-1}(\tau; \mu, \sigma, \alpha, \nu)\right)^2$$

By estimating the shape parameters $(\mu, \sigma, \alpha, \nu)$, we reconstruct the conditional distribution function of GDP growth. This reconstruction allows us to derive conditional probability statements. These statements provide a detailed understanding of the distributional characteristics of future GDP growth under varying financial and economic conditions.

The quantile score is chosen as the key metric for the evaluation. It allows validation based on selected quantiles. The quantile score is a loss function that penalizes deviations from the target quantile, with different weights for overestimates and underestimates. It is asymmetric and focuses on minimizing the error for a specific quantile of the target distribution. For a quantile $\tau \in (0,1)$ the quantile score for a single observation is defined as:

$$\rho_\tau(u) = \begin{cases} \tau \cdot u, & if\ u \geq 0 \\ (\tau - 1) \cdot u, & if\ u < 0 \end{cases}$$

Where $u = y - \hat{y}_\tau$ is the difference between the observed value y and the predicted quantile $\hat{y}_\tau$. This approach is particularly relevant in the context of quantile regressions, where it is not possible to directly observe the quantiles being forecasted. As a result, traditional methods of validation, such as comparing predicted values to observed outcomes, cannot be applied. This limitation contrasts with point estimation, where the accuracy of forecasts can be directly assessed by comparing predicted values to actual observations. The quantile score, therefore, serves as a practical and robust alternative for evaluating the model's performance by focusing on the accuracy of predictions within the selected quantiles. The evaluation period is chosen between the first quarter of 2003 - as this is when the first observation of all variables is available - and the second quarter of 2023 - as this represents the most recent data of each time series at the start of the evaluation.[10]

Additionally, the continuous ranked probability score (CRPS) is considered as a secondary metric. The CRPS is a measure for evaluating the accuracy of probabilistic forecasts, as it assesses the difference between the predicted cumulative distribution function and the observed outcome. In contrast to the quantile score, this metric evaluates the entire distribution rather than focusing on specific points, thereby capturing the uncertainty inherent in probabilistic models. By incorporating the CRPS, the analysis ensures an additional check for robustness. Furthermore, it is examined whether the combination or weighted combination of different metrics influences the results. For the weighted combination of metrics, the various metrics are standardized to enable a consistent comparison. The results remain valid when combining and weighting the metrics.

The analysis of the quantile regressions is based on real-time data that is updated over time, reflecting the information available at each point and its subsequent revisions. The first estimation is conducted 10 years after the first observation. For this, the data of the German GDP from the Broad Macroeconomic Projection Exercise (BMPE)[11] for the respective period is estimated as the endogenous variable. The forecast data from the BMPE are used to evaluate the

[10] Please note that estimates and figures shown later in this paper are based on more recent data. That is, the data was updated since the evaluation, with the last data point in this study being March 2025.
[11] For more information on the BMPE, see e.g. Haertel et al. (2022).

models, utilizing the most up-to-date data available for policymakers. For each quarter, a separate estimation is calculated using the respective BMPE time series to account for the data available in that quarter. Starting from the first quarter of 2013, an estimation is calculated for each following quarter per model specification. Thus, a total of 42 models are estimated per model specification, each for 42 different periods.

The first category, financial stress indicators, includes variables that depict short-term financial market information and financial market stress.[12] The DAX Volatility Index (VDAX), the Financial Stress Indicator for Germany (FSI), the Composite Indicator of Financial Conditions for Germany (CIFC), the Country-Level Index of Financial Stress for Germany (CLIFS), the Country-Level Index of Financial Stress for the three largest European economies, and the Composite Indicator of Systemic Stress (CISS) are considered.[13]

In the category of early-warning or financial-cycle indicators, various variables are examined that contain information about the buildup of vulnerabilities in the financial system. Specifically, the financial cycle indicator, the early warning indicator, the credit-to-GDP gap, housing prices, the change in the stock of loans and securities to domestic companies, the change in the stock of loans and securities to domestic individuals and non-profit organizations, and the change in the stock of loans and securities to domestic companies, individuals, and non-profit organizations are considered.[14]

Under the category of external factors, variables are summarized that describe the developments of US financial markets with a potential impact on the German market. Specifically, the National Financial Conditions Index of the USA (NFCI), the Chicago Board Options Exchange Volatility Index (VIX), and the Spillover Indicator are examined. Real economic indicators include variables that contain information about the expectations of producers and consumers as well as the actual production of the industry. Specifically, a survey-based sentiment indicator of the manufacturing sector in Germany, a survey-based sentiment indicator of consumers in Germany, and industrial production are considered.

The evaluated model should primarily enable robust estimation of downside risk. The quantile score is used as the metric for comparing different models. Both the quantile score for the 5% quantile and for the 10% quantile are calculated as an additional check for robustness. The evaluated model should also provide plausible estimates during the period of the global financial crisis. For this purpose, the quantile score for an in-sample forecast for the period of the global financial crisis is considered, where the global financial crisis is defined as the period between the first quarter of 2007 and the fourth quarter of 2009. For the in-sample forecast,

[12] Please see the appendix for more information on the variables used in the evaluation.
[13] For more information on the financial stress indicators employed here, see Metiu (2022, 2024), Duprey and Klaus (2017), Duprey et al. (2017), and Holló et al. (2012).
[14] For more information on financial cycle indicators and early warning models, see e.g. Beutel et al. (2019), Hiebert et al. (2018) and Schüler et al. (2015, 2017).

the coefficients of the most recent estimation are multiplied by the values of the quarters during the global financial crisis, summed according to the model specification, and the determined value is treated as a potential realization.

3 Comparison of model specifications

For the systematic comparison of models, the number of variables considered is reduced initially, as each additional variable within a category exponentially increases the number of models to evaluate. We specifically reduce the number of real economic indicators and external factors. These variables provide a natural starting point for model selection. They represent fundamental macroeconomic conditions and external developments. When combined with lags of GDP growth, they serve as an economic benchmark. This benchmark reflects baseline economic dynamics without financial stress. It also enables the assessment of the additional predictive value of financial variables. By initially evaluating models that vary only in real economic and external variables, we set a stable reference point. The specifications that include the sentiment indicator of consumers, industrial production, and the spillover indicator as explanatory variables show higher loss values of the quantile score compared to the specifications that consider the business confidence indicator of the manufacturing sector, NFCI, and VIX. Accordingly, for the systematic evaluation, the variables of the business confidence, NFCI, and VIX are further examined.

The systematic evaluation then examines model specifications of all remaining variables in all possible combinations. An autoregressive model of second order (i.e., an AR(2) model) is chosen as the starting point. Accordingly, the endogenous variable is explained by its prior values of the two preceding periods. In the next step, the AR(2) model is extended by the business confidence indicator of the manufacturing sector. Subsequently, estimations are calculated that consider two lagged values of the endogenous variable, the business confidence indicator of the manufacturing sector plus one variable from another category as explanatory variables. This is followed by an extension where a combination of two other categories is chosen, considering all possible combinations from the two other categories. In the final stage, the model is extended with a combination of three other categories. This approach allows the systematic examination of all possible combinations of variables, with a total of 200 model specifications being estimated.

The systematic calculation of all possible model specifications allows the sorting of models based on the quantile score, where the models are sorted in ascending order of the respective loss values. For the interpretation of the loss values, the AR(2) model and the AR(2) model plus NFCI are set as benchmark models. These benchmarks represent a purely autoregressive baseline and a minimal extension that incorporates international financial conditions. The

AR(2) specification captures the serial dependence in GDP growth. Adding the NFCI evaluates the marginal impact of U.S. financial conditions without including more complex domestic indicators. These benchmarks clarify the additional value of broader financial stress and sentiment measures in more comprehensive models. The assessment of the models is then based on the quantile score over the entire period, as well as the quantile score of the in-sample forecast for the global financial crisis.

The preliminary consideration of the quantile score shows significantly increased values due to the COVID period. Since the loss values of the quantile score exhibit unusually high values, the values of the quantile score are considered over the entire period. This consideration reveals that the high values of the quantile score are due to the periods of the COVID pandemic. Since the COVID pandemic does not represent a crisis resulting from endogenous events in the financial system but is classified by the ESRB crisis database as a unique "exogenous health crisis", including the COVID period would primarily evaluate a model that explains the downside risks during such an external shock.[15] Since the evaluated model aims to represent the downside risk for overall economic production in relation to financial developments, the COVID pandemic period is removed from the evaluation period.[16]

The 200 models are estimated excluding the COVID period and subsequently ranked by their quantile scores. The evaluation concentrates mainly on the 5% and 10% quantile scores during the normal period, with particular attention also given to in-sample forecasting performance. Additionally, the CRPS is used to assess the overall plausibility of the models. Models that demonstrate strong performance across these metrics are selected for more detailed analysis, regardless of the number of variables they include. First, the sign of the coefficients is checked to see if it aligns with an economically plausible impact on overall economic production. Second, the value of the coefficients over the iterative estimations is considered. It is examined whether the value of the coefficient remains stable over time or undergoes strong variations. The examination of the coefficients over time shows the fundamental tendency that the coefficients of the 5% quantile exhibit significantly stronger fluctuations than the coefficients of the 10% quantile. Additionally, the sign of the coefficients of the 5% quantile is often not stable. Therefore, a consistent interpretation of the impact of individual variables is not possible at all times. Accordingly, the focus of this investigation is primarily on the 10% quantile.

4 Result of the evaluation

The evaluation leads to the adoption of the following model specification, which is particularly suitable and robust for forecasting the quantiles of annualized GDP growth ($\widehat{GDPgrowth}$). The

[15] For more information on the COVID pandemic, see e.g. Padhan and Prabheesh (2021).
[16] Specifically, the values from the first quarter of 2020 to the second quarter of 2021 are removed from the time series.

model is specified as:

$$\widehat{GDPgrowth}_{t+1} = const. + GDPgrowth_t + GDPgrowth_{t-1} + BCI_t + CLIFS_t + NFCI_t$$

where the explanatory variables include the business confidence indicator of the manufacturing sector (BCI), the Country-Level Index of Financial Stress for Germany ($CLIFS$), as well as the National Financial Conditions Index of the USA ($NFCI$), in addition to lagged values of German GDP growth.

Figure 1 shows that this model specification ranks among the top 6% at the 5% quantile and the top 2.5% at the 10% quantile of all examined models with the lowest quantile score. Therefore, this model is very well suited to estimate the 5% and 10% quantile of overall economic development compared to the majority of other models. For the in-sample forecast of the global financial crisis, this model ranks among the top 7% at the 5% quantile and the top 9% at the 10% quantile of all examined models with the lowest quantile score. Thus, the model is also very well suited to estimate the 5% and 10% quantile of overall economic production during the global financial crisis compared to the other examined models. In comparison, the benchmark AR(2) model ranks among the top 26% at the 5% quantile and the top 32% at the 10% quantile of all examined models with the lowest quantile score.

For the in-sample forecast, the model ranks among the top 21% at the 5% quantile and the top 16% at the 10% quantile. The benchmark AR(2) + NFCI model ranks among the top 15% at the 5% quantile and the top 16% at the 10% quantile of all examined models with the lowest quantile score. For the in-sample forecast of the global financial crisis, this model ranks among the top 1% at the 5% quantile but only among the top 32% at the 10% quantile. Overall, the evaluated model has significantly lower loss values, and is better able to estimate the 5% and 10% quantile of overall economic production than the benchmark models without additional explanatory variables. While different metrics might in general point to different model specifications being appropriate, this model appears to provide the optimal structure for estimating downside risk both during normal periods and the 2007/2008 Global Financial Crisis, demonstrating robustness over time and across multiple metrics, while maintaining plausible economic relationships between the explanatory variables and German GDP.

Figure 1: Boxplot – Comparison Quantile Score - 5% Quantile and 10% Quantile

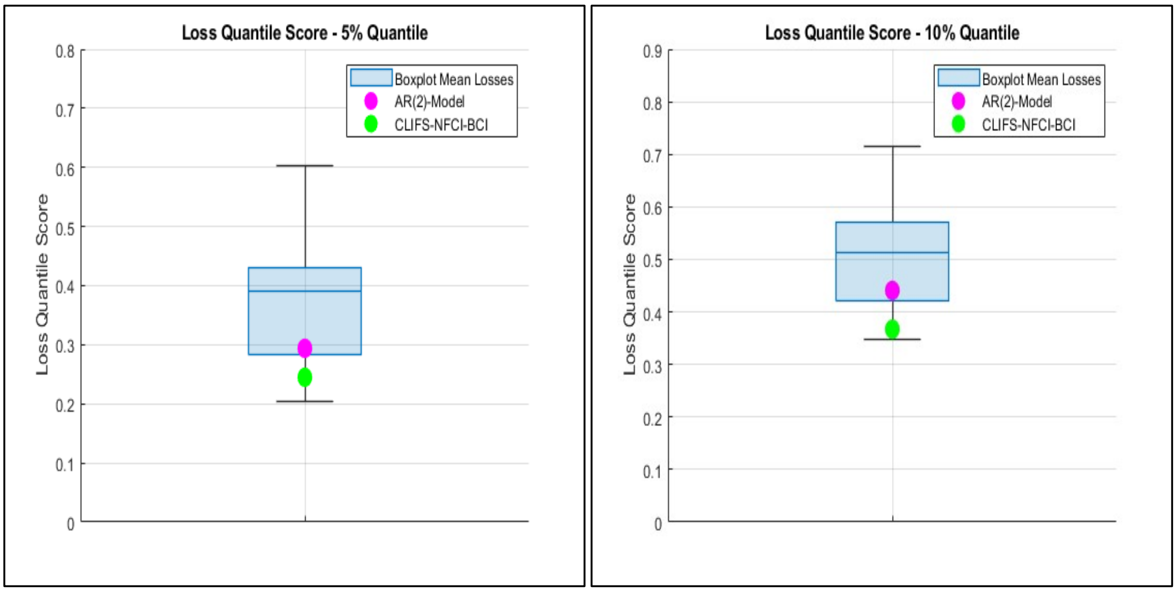
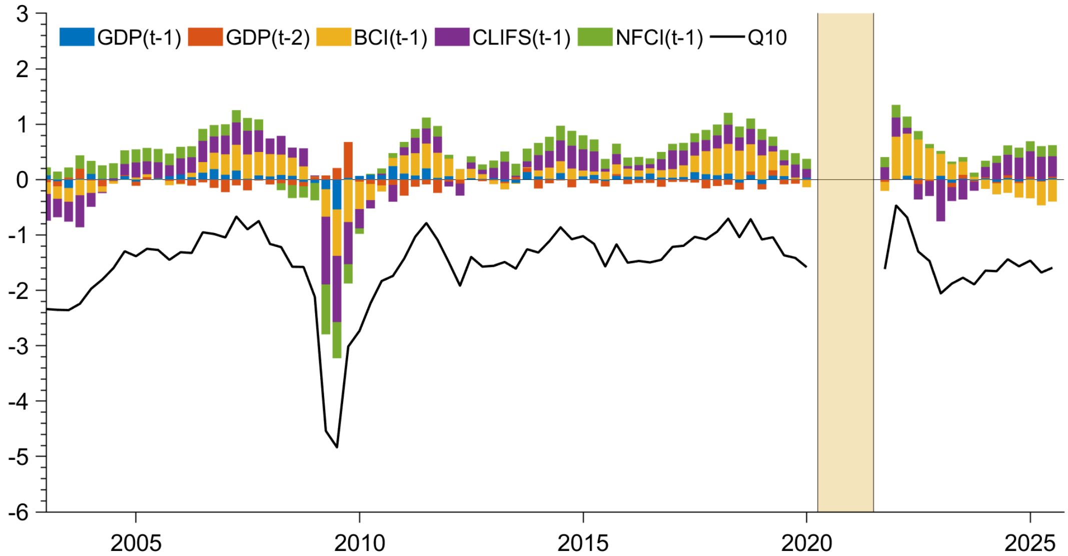


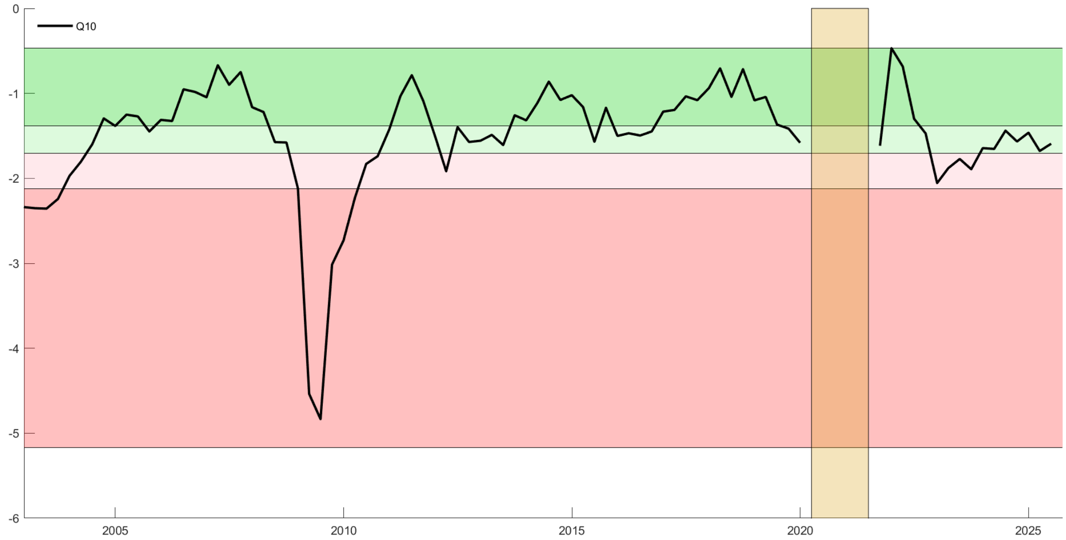
Figure 2 illustrates the impact of the variables on the 10% quantile of the one-quarter-ahead forecast of German GDP growth, capturing how downside risks evolve over time. While the contribution of these variables to tail risk fluctuates over time, the direction of the variables' effect on German GDP growth remains stable. For instance, during episodes of heightened financial stress—such as the 2007/2008 Global Financial Crisis—financial variables like CLIFS and NFCI strongly amplified downside risks. In more recent years, low levels of financial stress, as measured by these indices, have helped mitigate tail risks. Conversely, economic sentiment, as captured by the BCI, has emerged as a more significant driver of downside risk in recent years. Overall, the findings reveal that tail risks to German GDP growth estimated from financial conditions and business sentiment have been moderate over the past two years.

Figure 2: The evaluated GaR – model at the 10% quantile



To better judge estimates relative to historical norms, Figure 3 shows the historical 10% quantiles of the estimated model.[17] The colours in Figure 3 show the ranges of best (i.e., in green) and worst (i.e., in red) outcomes of the predictions over the time period. Given the current financial and economic conditions, the GaR model predicts less severe outcomes for the second quarter of 2025 (i.e., the light green color). Estimated outcomes are more favourable than, for example, in the past few years or during the 2007/2008 Global Financial Crisis. However, the estimates are not as positive as during the time periods directly before and after the COVID crisis.

Figure 3: The evaluated GaR – Historical quantiles

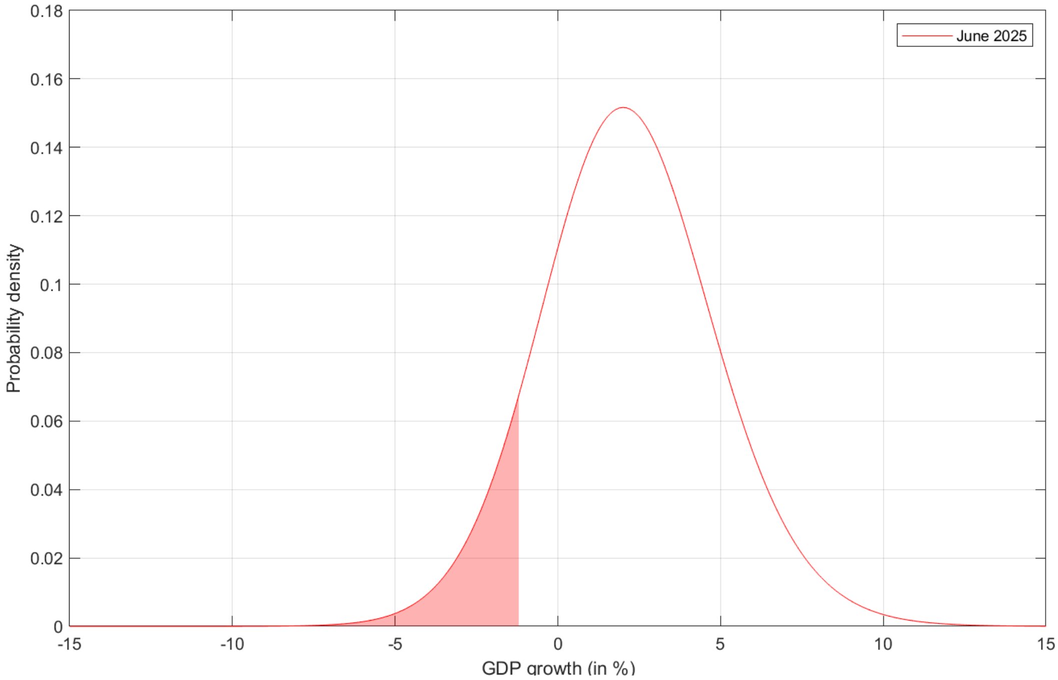


With the evaluated model, the conditional skewed t-distribution can be calculated and graphically represented. The conditional t-distribution presented in Figure 4, shows the estimated annualized change in German GDP on the x-axis, with the probability density being plotted on the y-axis. The t-distribution shows the relationship between the estimated probability and the change in German GDP growth for the second quarter of 2025.

In addition, the conditional t-distribution can be calculated at different points in time to illustrate the change in the forecast. By comparing the change in the conditional distribution at different points in time, the development of downside risk can be examined. Figure 5 compares the distribution for the 2025 Q2 estimate to the distribution for the 2023 Q2 as well as to the GFC (2009 Q1) estimate. Figure 5 points to a decrease of tail risk for the German economy between 2023 and 2025. That is, the improvement of financial conditions during this time period reduced the left skewness of the distribution, and hence the probability of severe negative outcomes. The probability of a severe recession in Germany has therefore decreased.

[17] The values of the historical quantiles (i.e., the 0, 0.25, 0.5, 0.75, and 1 quantile) are -5.1697, -2.1242, -1.7065, -1.3826, and -0.4684, respectively. Please note that, the historical quantiles are calculated over the time period from June 1971 to June 2025. Please see the appendix for more information.

Figure 4: Distribution of GDP growth rates in Germany – Estimates for 2025 Q2

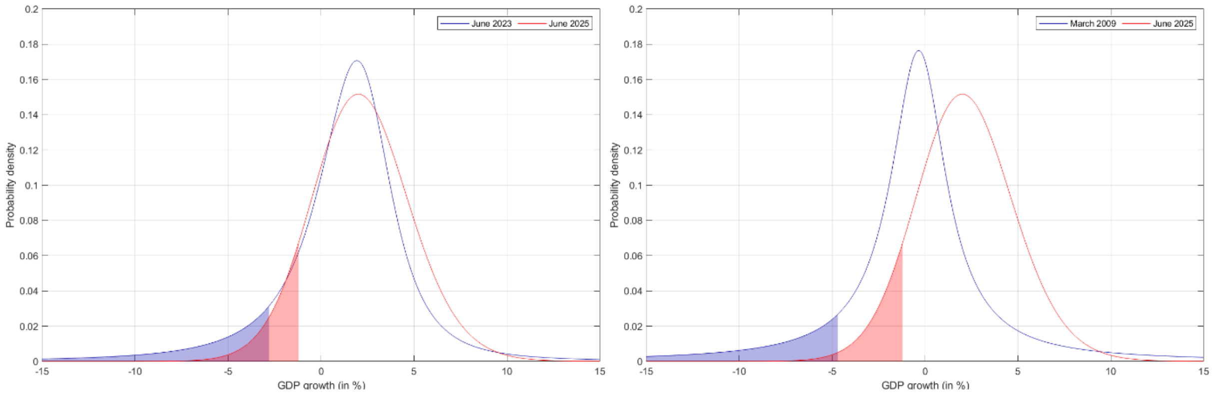


However, the median estimate for German growth appears comparable to 2023, with the centres of the respective distributions shown in Figure 5A being approximately at the same GDP growth levels. Recent market turmoil is therefore not represented in the US NFCI as well as in the other indicators for financial stress and business confidence, since the last data point is from March 2025. Therefore, in addition, Figure 5B compares the current situation to the 2007/2008 Global Financial Crisis, more specifically, to the 2009 Q1 forecast. The distribution of GDP growth during the GFC, which was characterized by severe financial stress and economic turmoil, shows a clear left skewness when compared to the current 2025 Q2 forecast.

Figure 5: Distribution of GDP growth rates in Germany – Historical comparisons

A: 2025 Q2 vs. 2023 Q2

B: 2025 Q2 vs. GFC (2009 Q1)



5 Conclusion

Summarizing, this paper describes the approach to evaluating a Growth-at-Risk model for German GDP growth, focusing on the analysis of tail risk in relation to financial developments. Through a systematic comparison of 200 model specifications, the findings indicate that the Country-Level Index of Financial Stress for Germany, the National Financial Conditions Index of the USA, a sentiment indicator of German companies and the GDP growth of the two preceding period are appropriate explanatory variables. The evaluation also reveals that the coefficients of the 10% quantile generally exhibit less variation compared to those of the 5% quantile. Hence, the 10% quantile is more robust for estimating the downside risk of German GDP. In addition, data for the COVID period are deleted from the time series, since the COVID pandemic does not represent a crisis resulting from endogenous events in the financial system but an exogenous health crisis. The estimation results for GDP growth show the impact of financial conditions on downside risk. Lower financial stress, as measured by the CLIFS and NFCI indices, contributes to the reduction of these risks, while increased economic sentiment, as measured by the business confidence indicator, has heightened downside vulnerabilities in recent years. Overall, the model identifies a decline in tail risk for German GDP growth over the past two years.

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A Variables used in the evaluation

Economic Indicators	Early Warning/ Financial Cycle Indicators	Financial Stress Indicators	External Factors
Business Confidence	Financial Cycle Indicator for Germany	DAX Volatility Index (VDAX)	National Financial Conditions Index of the USA (NFCI)
Consumer Confidence	Early Warning Indicator for Germany	Financial Stress Indicator for Germany (FSI)	Chicago Board Options Exchange Volatility Index (VIX)
Industrial Production	Residential Real Property Prices	Composite Indicator of Financial Conditions for Germany (CIFC)	Spillover Indicator
	Change in the Stock of Loans and Securities to Domestic Companies	Country-Level Index of Financial Stress for Germany (CLIFS)	
	Change in the Stock of Loans and Securities to Domestic Individuals and Non-Profit Organizations	Composite Indicator of Systemic Stress (CISS)	
	Change in the Stock of Loans and Securities to Domestic Companies, Individuals, and Non-Profit Organizations	Country-Level Index of Financial Stress for the three largest European economies (CLIFS_Euro3)	
	Credit-to-GDP Gap		

B The evaluated GaR and historical quantiles – 1971 to 2025