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Forecasting with Log-Linear (S)VAR Models: Incorporating Annual Growth Rate Conditions*

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Abstract

This note explores conditional forecasting under conditions on annual growth rates, where variables enter a (possibly structural) vector autoregressive (VAR) model in logarithms or logarithmic first differences. For example, imposing conditions on the annual growth rate of quarterly real GDP modeled in logarithms is challenging because annual growth rates are nonlinear functions of the log variables. We address this by approximating the annual growth rate with a linear function of the model variables, enabling the use of standard conditional forecasting methods. An approximation error arises since the condition is not imposed directly; to mitigate this, we iteratively adjust the condition until the error is acceptable. We provide MATLAB companion code that also accepts other types of conditions: (1) conditions on the path of variables entering the VAR, (2) conditions on the path of structural shocks, and (3) conditions on sums of successive variable observations.

Keywords: conditional forecasting; annual growth rate constraints; log-linear approximation; structural vector autoregression.

JEL classification: C32; C53; E17.

*Discussion Papers represent the authors' personal opinions and do not necessarily reflect the views of the Deutsche Bundesbank or the Eurosystem.

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1 Introduction

In this note, we consider conditional forecasting for a special conditions type. Specifically, we impose conditions on annual growth rates, i.e. the rate of growth between one calendar year and the next, of variables entering a vector autoregressive (VAR) model as log quarterly (or monthly) levels or first differences thereof.

The related literature on conditional forecasting in VARs includes, among others, Waggoner and Zha (1999), who consider conditional forecasting in a fully Bayesian manner, Jarocinski (2010), who extends the latter approach, Camba-Mendez (2012), who presents a non-Bayesian implementation of conditional forecasting in structural VARs using Kalman filtering techniques, and Antolin-Diaz, Petrella, and Rubio-Ramirez (2021), who study conditional forecasting in structural VARs. Similar approximations of annual growth rates have been proposed in the modelling of fixed-event survey forecasts, see Patton and Timmermann (2011) or Krüger and Plett (2024). A related problem arises in the temporal disaggregation of low-frequency indicators in mixed frequency models such as Mariano and Murasawa (2003) and Proietti and Moauro (2006).

One possible application of our approach is conditional forecasting under the assumption that real GDP, consumer prices, and other headline indicators evolve according to an official forecast. The problem is that such official forecasts are often available only for annual variables, i.e., at a frequency lower than that of our models. An example of such headline forecasts is the Broad Macroeconomic Projection Exercise (BMPE) produced by the Eurosystem and ECB staff, in which some projections are published only for annual variables, see [ECB website](#).

This note has two main contributions: Firstly, it considers a special type of condition that, to our knowledge, has not been considered before in the context of conditional forecasting. The second contribution is a comprehensive collection of MATLAB code that implements conditional forecasting in an accessible way.¹

2 Linear approximation

We want to impose an annual growth rate condition of the type

$$\frac{\sum_{t \in \mathbb{Y}_\tau} z_t}{\sum_{t \in \mathbb{Y}_{\tau-1}} z_t} = 1 + g, \quad (1)$$

¹See <https://github.com/markusrothbbk/conditional-forecasts-with-log-linear-svar>.

where $\mathbb{Y}_\tau$ is the index set of periods corresponding to the year τ ,² z_t is the period- t level of the variable of interest, and g is the respective net growth rate.

Our problem arises from the fact that z enters the VAR model in log levels or in log first differences, i.e. we have a VAR

$$x_t = Bx_{t-1} + R\varepsilon_t, \quad (2)$$

where either $\ln z_t \in x_t$ or $\Delta \ln z_t \in x_t$. Definitions: x_t is a $k \times 1$ vector of macroeconomic variables, B and R are $k \times k$ parameter matrices, $\varepsilon_t \sim N(\mathbf{0}_{(k \times 1)}, \mathbf{I}_{(k \times k)})$, $\mathbf{0}_{(k \times 1)}$ is a $k \times 1$ vector of zeros, and $\mathbf{I}_{(k \times k)}$ is a $k \times k$ identity matrix. The notation of equation (2), borrowed from [Camba-Mendez \(2012\)](#), allows the VAR to be structurally identified by the matrix R . For simplicity, we examine a VAR with a single lag of x_t and no constant. The extension to a VAR with p lags and a constant is straightforward. The code accompanying this note considers the general case.

Our approach is to make a log-linear approximation to the annual growth rate so that it can be expressed as a linear function of the model variables. This step allows us to apply standard conditional forecasting methods. For this purpose, we first express the annual growth rate condition (1) in logarithms:

$$\ln \left(\sum_{t \in \mathbb{Y}_\tau} z_t \right) - \ln \left(\sum_{t \in \mathbb{Y}_{\tau-1}} z_t \right) = \ln(1 + g). \quad (3)$$

Next, we define the function $f(\{z_t\}_{t \in \mathbb{Y}_\tau}, \{z_t\}_{t \in \mathbb{Y}_{\tau-1}})$ as the left-hand side of equation (3), i.e.

$$f(\{z_t\}_{t \in \mathbb{Y}_\tau}, \{z_t\}_{t \in \mathbb{Y}_{\tau-1}}) := \ln \left(\sum_{t \in \mathbb{Y}_\tau} z_t \right) - \ln \left(\sum_{t \in \mathbb{Y}_{\tau-1}} z_t \right). \quad (4)$$

A first-order Taylor series expansion of equation (4) yields the approximation that we use

²For example, for quarterly data, the index set $\mathbb{Y}_{2023}$ will include the quarterly time indices corresponding to year 2023, i.e. $\mathbb{Y}_{2023} = \{2023:Q1, 2023:Q2, 2023:Q3, 2023:Q4\}$. Here the term “index set” refers to a collection of time indices that identify specific quarters within a given year.

to impose annual growth rate conditions:

$$\begin{aligned}
& f\left(\{z_t\}_{t \in \mathbb{Y}_\tau}, \{z_t\}_{t \in \mathbb{Y}_{\tau-1}}\right) \\
& \approx f\left(\{z_t = \bar{z}_\tau\}_{t \in \mathbb{Y}_\tau}, \{z_t = \bar{z}_{\tau-1}\}_{t \in \mathbb{Y}_{\tau-1}}\right) + \sum_{t \in \mathbb{Y}_\tau} \frac{\partial f(\cdot, \cdot)}{\partial \ln z_t} \Big|_{z_t = \bar{z}_\tau} (\ln z_t - \ln \bar{z}_\tau) \\
& \quad - \sum_{t \in \mathbb{Y}_{\tau-1}} \frac{\partial f(\cdot, \cdot)}{\partial \ln z_t} \Big|_{z_t = \bar{z}_{\tau-1}} (\ln z_t - \ln \bar{z}_{\tau-1}) \\
& = \ln n \bar{z}_\tau - \ln n \bar{z}_{\tau-1} + \frac{\bar{z}_\tau}{\sum_{t \in \mathbb{Y}_\tau} \bar{z}_\tau} \sum_{t \in \mathbb{Y}_\tau} (\ln z_t - \ln \bar{z}_\tau) \\
& \quad - \frac{\bar{z}_{\tau-1}}{\sum_{t \in \mathbb{Y}_{\tau-1}} \bar{z}_{\tau-1}} \sum_{t \in \mathbb{Y}_{\tau-1}} (\ln z_t - \ln \bar{z}_{\tau-1}) \\
& = \frac{1}{n} \sum_{t \in \mathbb{Y}_\tau} \ln z_t - \frac{1}{n} \sum_{t \in \mathbb{Y}_{\tau-1}} \ln z_t,
\end{aligned} \tag{5}$$

where we have obtained the partial derivative $\frac{\partial f(\cdot, \cdot)}{\partial \ln z_t}$ by the chain rule as $\frac{\partial f(\cdot, \cdot)}{\partial \ln z_t} = \frac{\partial f(\cdot, \cdot)}{\partial z_t} \frac{\partial z_t}{\partial \ln z_t}$, and $\frac{\partial z_t}{\partial \ln z_t} = \left(\frac{\partial \ln z_t}{\partial z_t}\right)^{-1} = z_t$ by the inverse function rule. In equation (5), the parameter n is the number of elements in $\mathbb{Y}_\tau$ or $\mathbb{Y}_{\tau-1}$, i.e. $n = 4$ for quarterly data and $n = 12$ for monthly data.

According to equation (5), we can approximate the annual growth rate by the difference of the annual average log levels of z_t in years τ and $\tau - 1$. If we rearrange the terms, we see that the approximation is equivalent to $\frac{1}{n} \sum_{t \in \mathbb{Y}_\tau} (\ln z_t - \ln z_{t-n})$. In the case of quarterly data that means that we approximate the annual growth rate by averaging across 4-quarter-log-growth rates.

Using this linear approximation to the annual growth rate, we impose the condition

$$\frac{1}{n} \sum_{t \in \mathbb{Y}_\tau} \ln z_t - \frac{1}{n} \sum_{t \in \mathbb{Y}_{\tau-1}} \ln z_t = \ln(1 + g). \tag{6}$$

Since this condition is linear, we can use standard methods to obtain conditional forecasts. Specifically, if we define

$$X_{T+1:T+H} := \begin{bmatrix} x'_{T+1}, & \cdots, & x'_{T+H} \end{bmatrix}'$$

as the path of dependent variables in equation (2) for the period $T + 1$ to $T + H$, and we impose the linear conditions as

$$PX_{T+1:T+H} = \rho, \tag{7}$$

then it holds that

$$\begin{bmatrix} X_{T+1:T+H} \\ PX_{T+1:T+H} \end{bmatrix} \middle| x_T, \dots, x_1; \theta \sim N \left(\begin{bmatrix} \mu \\ P\mu \end{bmatrix}; \begin{bmatrix} \Sigma & \Sigma P' \\ P\Sigma & P\Sigma P' \end{bmatrix} \right).$$

Based on the properties of the multivariate normal distribution, we know that the conditional predictive density is equal to:

$$\begin{aligned} X_{T+1:T+H} | PX_{T+1:T+H} = \rho; x_T, \dots, x_1; \theta \\ \sim N \left(\mu + \Sigma P' [P\Sigma P']^{-1} [\rho - P\mu], \Sigma - \Sigma P' [P\Sigma P']^{-1} P\Sigma \right), \end{aligned} \quad (8)$$

where μ and Σ are functions of the history $x_t, \dots, x_T$ and the VAR parameters collected in $\theta = \{B, R\}$. Obviously, one way to acquire conditional forecasts is to articulate the condition by appropriately selecting the parameter P and ρ , as demonstrated in equation (7), and obtaining the distributional parameters according to equation (8). Another approach is to employ Kalman filtering techniques, as described in Appendix A. A potential advantage of the Kalman filter approach is increased computational efficiency because it works with smaller matrices.

Next, we show how to deal with the case where z_t enters the VAR in log first differences, i.e. as $\Delta \ln z_t$. To do this, we transform the left-hand side of equation (6) into a function of $\Delta \ln z_t$:

$$\begin{aligned} \frac{1}{n} \sum_{t \in \mathbb{Y}_\tau} \ln z_t - \frac{1}{n} \sum_{t \in \mathbb{Y}_{\tau-1}} \ln z_t \\ &= \frac{1}{n} \sum_{t \in \mathbb{Y}_\tau} (\ln z_t - \ln z_{t-n}) \\ &= \frac{1}{n} \sum_{t \in \mathbb{Y}_\tau} (\Delta \ln z_t + \Delta \ln z_{t-1} + \dots + \Delta \ln z_{t-n+1}) \\ &= \frac{1}{n} \Delta \ln z_{T+h} + \frac{2}{n} \Delta \ln z_{T+h-1} + \dots + \frac{n-1}{n} \Delta \ln z_{T+h-n+2} \\ &\quad + \Delta \ln z_{T+h-n+1} + \frac{n-1}{n} \Delta \ln z_{T+h-n} + \dots + \frac{1}{n} \Delta \ln z_{T+h-2n+1} \end{aligned} \quad (9)$$

where $T+h = \max(\mathbb{Y}_\tau)$ is the largest time index in the index set $\mathbb{Y}_\tau$. The right-hand side of equation (9) tells us how to weight the log first differences $\Delta \ln z_t$ to get the same approximation to the annual growth rate as in equation (6). [Blunier and Hepenstrick \(2022\)](#) use a very similar approximation to estimate quarterly GDP growth forecasts implicit in annual growth rate projections. Appendix B shows how we can treat this case using Kalman filtering techniques.

3 Reducing the approximation error

Suppose $\ln \hat{z}_t$ is the conditional forecast of $\ln z_t$, which satisfies the approximate annual growth rate condition $\frac{1}{n} \sum_{t \in \mathbb{Y}_\tau} \ln \hat{z}_t - \frac{1}{n} \sum_{t \in \mathbb{Y}_{\tau-1}} \ln \hat{z}_t = \ln(1 + g)$. Then we will generally make an approximation error in the sense that our estimates do not satisfy the exact condition:

$$\frac{\sum_{t \in \mathbb{Y}_\tau} \hat{z}_t}{\sum_{t \in \mathbb{Y}_{\tau-1}} \hat{z}_t} \neq 1 + g.$$

The following algorithm iteratively adjusts the condition according to the approximation error:

1. Obtain the first set of conditional forecasts $\left\{ \hat{z}_t^{(it=1)} \right\}_{t=T+1 \dots T+H}$ using the condition $\frac{1}{n} \sum_{t \in \mathbb{Y}_\tau} \ln z_t - \frac{1}{n} \sum_{t \in \mathbb{Y}_{\tau-1}} \ln z_t = \ln(1 + g)$.
2. Compute the approximation error to the annual growth rate condition: $\xi^{(it=1)} = \frac{\sum_{t \in \mathbb{Y}_\tau} \hat{z}_t^{(it=1)}}{\sum_{t \in \mathbb{Y}_{\tau-1}} \hat{z}_t^{(it=1)}} - (1 + g)$.
3. While $|\xi^{(it)}| > \kappa$, repeat the next two steps.
4. Obtain a new set of conditional forecasts $\left\{ \hat{z}_t^{(it+1)} \right\}_{t=T+1 \dots T+H}$ using the condition $\frac{1}{n} \sum_{t \in \mathbb{Y}_\tau} \ln z_t - \frac{1}{n} \sum_{t \in \mathbb{Y}_{\tau-1}} \ln z_t = g^{(it+1)}$ where $g^{(it+1)} = g^{(it)} - \alpha \xi^{(it)}$, $\alpha \in (0, 1)$, and $g^{(1)} = \ln(1 + g)$.
5. Compute new approximation error: $\xi^{(it+1)} = \frac{\sum_{t \in \mathbb{Y}_\tau} \hat{z}_t^{(it+1)}}{\sum_{t \in \mathbb{Y}_{\tau-1}} \hat{z}_t^{(it+1)}} - (1 + g)$.

Step 4 of the algorithm means that the approximation-based target, $g^{(it+1)}$, is decreased (i.e. $g^{(it+1)} < g^{(it)}$) if the previous forecast was too high (i.e. $\xi^{(it)} > 0$), and vice versa if the previous forecast was too low. We have had good experience with setting $\kappa = 10^{-6}$ and $\alpha = 0.5$. With these parameter values, the approximation error to the exact condition is at most 10^{-4} percentage points, which we consider sufficiently accurate for our applications.

4 Application

This application explores the circumstances under which one can expect large approximation errors if the algorithm of section 3 is not used. To do this, we use simulated data from a 3-variable VAR, see Appendix C. In turn, we generate forecasts conditional on the exponential value³ of a variable having a $g \in \{\pm 0\%, \pm 10\%, \pm 30\%, \pm 50\%\}$ lower/higher value

³I.e. we assume that the variable enters the VAR in logarithmic values, and that the condition refers to the (untransformed) levels of the variable.

between $T + 1$ to $T + 4$ on average than between $T - 3$ to T . Here, we use ordinary least squares estimates of the VAR coefficients $\{B, R\}$. Table 1 shows the main results: (i) the value g of the condition we impose, (ii) the growth rate $(\hat{z}_{T+1} + \dots + \hat{z}_{T+4}) / (\hat{z}_{T-3} + \dots + \hat{z}_T) - 1$ obtained by computing the conditional forecasts $\hat{z}_t$ as in section 2 without the iteration of section 3, and (iii) the same growth rate if we apply the iteration of section 3. The main observation is that the approximation errors are larger when the imposed growth rate g is large in absolute value and they become negligibly small as the absolute value of g approaches zero.⁴

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⁴The companion code file also considers the case where we impose the same conditions on a first-difference VAR. The results are virtually identical.

Tables

Table 1: Application on simulated data.

imposed condition g	value in conditional forecast without iteration	value in conditional forecast with iteration
-0.5	-0.47385	-0.50000
-0.3	-0.29011	-0.30000
-0.1	-0.09871	-0.01000
0.0	-0.00003	0.00000
0.1	0.10063	0.10000
0.3	0.30780	0.30000
0.5	0.52259	0.50000

Notes: (i) values rounded to 5 digits after the decimal point; (ii) reading example: imposed condition of -0.5 is equivalent to -50% growth.

Appendix

A State-space system when variables are in log levels

In what follows, we use both the notation and the main results described in [Camba-Mendez \(2012\)](#) with minor extensions to account for our special case. To produce conditional forecasts under the annual growth rate condition, we rewrite the VAR in state-space form as

$$\begin{aligned}y_t &= C s_t \\s_t &= A s_{t-1} + u_t,\end{aligned}\tag{10}$$

where

$$\begin{aligned}
y_t &= \begin{bmatrix} x_t \\ \varepsilon_{t+1} \\ w_t \end{bmatrix}; \quad w_t = \frac{(x_t + \dots + x_{t-n+1}) - (x_{t-n} + \dots + x_{t-2n+1})}{n}; \\
C &= \begin{bmatrix} \mathbf{0}_{(k \times k)} & \begin{bmatrix} \mathbf{I}_{(k \times k)} & \mathbf{0}_{(k \times 2nk-k)} \\ \mathbf{0}_{(k \times k)} & \mathbf{0}_{(k \times 2nk-k)} \end{bmatrix} \\ \mathbf{I}_{(k \times k)} & \\ \mathbf{0}_{(k \times k)} & n^{-1} \begin{bmatrix} \mathbf{I}_{(k \times k)} & \dots & \mathbf{I}_{(k \times k)} & -\mathbf{I}_{(k \times k)} & \dots & -\mathbf{I}_{(k \times k)} \end{bmatrix} \end{bmatrix}; \\
s_t &= \begin{bmatrix} \varepsilon_{t+1} \\ x_t \\ \vdots \\ x_{t-2n+1} \end{bmatrix}; \quad A = \begin{bmatrix} \mathbf{0}_{(k \times k)} & \mathbf{0}_{(k \times 2nk)} \\ R & \begin{bmatrix} B & \mathbf{0}_{(k \times 2nk-k)} \end{bmatrix} \\ \mathbf{0}_{(2nk-k \times k)} & \begin{bmatrix} \mathbf{I}_{(2nk-k \times 2nk-k)} & \mathbf{0}_{(2nk-k \times k)} \end{bmatrix} \end{bmatrix}; \\
u_t &= \begin{bmatrix} \varepsilon_{t+1} \\ \mathbf{0}_{(2nk \times 1)} \end{bmatrix}; \quad \Sigma_u = \mathbb{E}[u_t u_t'] = \begin{bmatrix} \mathbf{I}_{(k \times k)} & \mathbf{0}_{(k \times 2nk)} \\ \mathbf{0}_{(2nk \times k)} & \mathbf{0}_{(2nk \times 2nk)} \end{bmatrix};
\end{aligned}$$

- We want to impose the condition $w_{T+h} = \ln(1+g)$ with $1 \leq h \leq H$.
- We compute the conditional forecast $E[y_{T+1}, \dots, y_{T+H} | y_1, \dots, y_T, w_{T+H} = \ln(1+g); \theta]$, using the [Ansley and Kohn \(1982\)](#) fixed interval smoother, assuming that the VAR parameters collected in θ are known. For a description of the fixed interval smoother algorithm, see section 2.3 of [Camba-Mendez \(2012\)](#).
- Table 3 describes which data on the observables we need to provide to the algorithm to obtain the conditional forecasts. For simplicity, the table assumes as an example a VAR with $k = 2$ variables and that the annual growth rate condition refers to the first variable in x_t at the horizon $T+h$. The data input includes the history of x_t from $t = T-1$ to T . In addition, we treat all elements of y_t from $t = T+1$ to $T+H$ as missing except for the value of w_t on which we want to impose the approximate annual growth rate condition.
- Note that for our purposes, in principle, it is not necessary to include ε_{t+1} in y_t , since we are not focusing on conditional predictions of ε_t , nor are we imposing conditions

Table 2: Data input to fixed interval smoother algorithm; case: log levels.

y'_t	$x_{1,t}$	$x_{2,t}$	$\varepsilon_{1,t}$	$\varepsilon_{2,t}$	$w_{1,t}$	$w_{2,t}$
y'_{T-1}	$x_{1,T-1}$	$x_{2,T-1}$	$\hat{\varepsilon}_{1,T-1}$	$\hat{\varepsilon}_{2,T-1}$	$\square$	$\square$
y'_T	$x_{1,T}$	$x_{2,T}$	$\square$	$\square$	$\square$	$\square$
y'_{T+1}	$\square$	$\square$	$\square$	$\square$	$\square$	$\square$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
y'_{T+h-1}	$\square$	$\square$	$\square$	$\square$	$\square$	$\square$
y'_{T+h}	$\square$	$\square$	$\square$	$\square$	$\ln(1+g)$	$\square$
y'_{T+h+1}	$\square$	$\square$	$\square$	$\square$	$\square$	$\square$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
y'_{T+H}	$\square$	$\square$	$\square$	$\square$	$\square$	$\square$

Missing values are indicated by a square (“ $\square$ ”).

on ε_t .

- Note that additional conditions can be imposed by specifying the corresponding elements in the data input, see table 3. The companion code allows various other conditions. These conditions often require a larger state-space system. For a discussion of well-defined conditional forecasts, see [Camba-Mendez \(2012\)](#), Section 2.2.

B State-space representation of VAR when variable z_t enters in log first differences.

In the following, we assume that the VAR is specified in first differences, i.e. equation (2) becomes

$$\Delta x_t = B\Delta x_{t-1} + R\varepsilon_t. \quad (11)$$

Beyond that, we specify a similar state-space model as in section A. The main difference is that x_t enter y_t in first differences, i.e. as $\Delta \ln x_t$, and that lower block of parameters in C has a different structure. The reason for this is that the approximation to annual growth rate constraint is different, see equation (9).

$$\begin{aligned} y_t &= Cs_t \\ s_t &= As_{t-1} + u_t, \end{aligned} \quad (12)$$

where

$$y_t = \begin{bmatrix} \Delta x_t \\ \varepsilon_{t+1} \\ w_t \end{bmatrix}; w_t = \frac{(x_t + \dots + x_{t-n+1}) - (x_{t-n} + \dots + x_{t-2n+1})}{n};$$

$$C = \begin{bmatrix} \mathbf{0}_{(k \times k)} & \begin{bmatrix} \mathbf{I}_{(k \times k)} & \mathbf{0}_{(k \times 2nk-2k)} \\ \mathbf{0}_{(k \times k)} & \mathbf{0}_{(k \times 2nk-2k)} \end{bmatrix} \\ \mathbf{I}_{(k \times k)} & \\ \mathbf{0}_{(k \times k)} & n^{-1} \begin{bmatrix} \mathbf{I}_{(k \times k)} & 2 \cdot \mathbf{I}_{(k \times k)} & \dots & n \cdot \mathbf{I}_{(k \times k)} & (n-1) \cdot \mathbf{I}_{(k \times k)} & \dots & \mathbf{I}_{(k \times k)} \end{bmatrix} \end{bmatrix};$$

$$s_t = \begin{bmatrix} \varepsilon_{t+1} \\ \Delta x_t \\ \vdots \\ \Delta x_{t-2n+2} \end{bmatrix}; A = \begin{bmatrix} \mathbf{0}_{(k \times k)} & \mathbf{0}_{(k \times 2nk-k)} \\ R & \begin{bmatrix} B & \mathbf{0}_{(k \times 2nk-2k)} \end{bmatrix} \\ \mathbf{0}_{(2nk-2k \times k)} & \begin{bmatrix} \mathbf{I}_{(2nk-2k \times 2nk-k)} & \mathbf{0}_{(2nk-2k \times k)} \end{bmatrix} \end{bmatrix};$$

$$u_t = \begin{bmatrix} \varepsilon_{t+1} \\ \mathbf{0}_{(2nk-k \times 1)} \end{bmatrix}; \Sigma_u = \mathbb{E}[u_t u_t'] = \begin{bmatrix} \mathbf{I}_{(k \times k)} & \mathbf{0}_{(k \times 2nk-k)} \\ \mathbf{0}_{(2nk-k \times k)} & \mathbf{0}_{(2nk-k \times 2nk-k)} \end{bmatrix};$$

Table 3: Data input to fixed interval smoother algorithm; case: log first differences

y'_t	$\Delta x_{1,t}$	$\Delta x_{2,t}$	$\varepsilon_{1,t}$	$\varepsilon_{2,t}$	$w_{1,t}$	$w_{2,t}$
y'_{T-1}	$\Delta x_{1,T-1}$	$\Delta x_{2,T-1}$	$\hat{\varepsilon}_{1,T-1}$	$\hat{\varepsilon}_{2,T-1}$	$\square$	$\square$
y'_T	$\Delta x_{1,T}$	$\Delta x_{2,T}$	$\square$	$\square$	$\square$	$\square$
y'_{T+1}	$\square$	$\square$	$\square$	$\square$	$\square$	$\square$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
y'_{T+h-1}	$\square$	$\square$	$\square$	$\square$	$\square$	$\square$
y'_{T+h}	$\square$	$\square$	$\square$	$\square$	$\ln(1+g)$	$\square$
y'_{T+h+1}	$\square$	$\square$	$\square$	$\square$	$\square$	$\square$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
y'_{T+H}	$\square$	$\square$	$\square$	$\square$	$\square$	$\square$

C Simulated data

We start by simulating $T = 200$ observations from the following stationary 3-variable VAR:

$$x_t = \begin{bmatrix} 0 \\ 0.0025 \\ 0.0025 \end{bmatrix} + \begin{bmatrix} 0.5 & 0 & 0 \\ 0.1 & 0.5 & 0.1 \\ 0.1 & 0.1 & 0.5 \end{bmatrix} x_{it-1} + \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} x_{it-2} + \begin{bmatrix} 0.001 & 0 & 0 \\ 0.001 & 0.0017 & 0 \\ 0.001 & 0.0006 & 0.0016 \end{bmatrix} \varepsilon_t,$$

where $\varepsilon_t \sim N(\mathbf{0}_{(3 \times 1)}, \mathbf{I}_{(3 \times 3)})$. Thus, the variables have some persistence and are related both through correlated disturbances and through non-zero off-diagonal values in the lag polynomial matrices. These simulated observations are assumed to represent (log) first differences; (ii) We generate (log) level series by setting meaningful initial levels and cumulating the simulated first difference observations. We apply the approximate annual growth rate condition to the third variable. For details see the companion code file `conditionalForecast_applicationNote.m`.