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Monetary policy and supply-side turnover

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Monetary Policy and Supply-Side Turnover*

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Abstract

The introduction of a firm or product life cycle into New Keynesian frameworks fundamentally alters the design of optimal monetary policy. Economic welfare and the Phillips curve then depend on the gap between inflation and a time-varying inflation target that arises endogenously from turnover. The inflation target is positive on average and shifts in response to productivity disturbances. As a result, steady-state price stability is no longer desirable and the dynamics make it optimal for monetary policy to "look through" certain productivity disturbances. The latter requires keeping nominal rates unchanged even though both output and inflation move. This complicates the empirical distinction between supply, demand, and policy shocks. Our results highlight that accounting for supply side turnover delivers a rich set of policy-relevant results for inflation targeting and shock identification.

JEL-Class.No.: E31, E32, E52, E61

Keywords: firm turnover, product turnover, optimal monetary policy, time-varying inflation target

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1 Introduction

Monetary policy analysis typically relies on economic models that assume a fixed set of firms or products. In modern economies, however, turnover is a ubiquitous feature: firms frequently enter and exit markets, and new products continuously replace old ones. Broda and Weinstein (2010) document, for example, high product turnover rates and Bils (2009) and Adam and Weber (2023) show that new products contribute substantially to measured inflation.

This chapter considers a tractable sticky-price model with supply-side turnover to analyze the monetary policy implications of supply-side turnover. Specifically, it derives the linear-quadratic approximation to the optimal monetary policy problem for a setting with turnover and shows how turnover fundamentally changes several canonical conclusions of monetary policy analysis derived from models without turnover. Although the setup can be interpreted as featuring either firm or product turnover, we focus on the firm turnover interpretation to simplify the exposition.

Firm turnover arises within our setup because firms face an exogenous idiosyncratic risk of becoming unproductive. The materialization of this risk causes them to exit the economy, upon which they are replaced by a new young firm. Firm technology is affected by cohort- and experience-specific productivity growth trends and is subject to three different kinds of supply shocks:

(i) *shocks to common productivity growth* that uniformly affect the productivity of all firms,

(ii) *cohort-productivity growth shocks* that affect the initial productivity of newly entering firm cohorts only, and

(iii) *experience-accumulation growth shocks* that alter the productivity of incumbent firms only.

Considering this setup in the presence of sticky prices, we derive the linearized New Keynesian Phillips curve and the quadratically approximated welfare objective. We show that (1) the welfare objective and the New Keynesian Phillips curve feature the gap between *inflation* and a *time-varying optimal inflation target*, instead of simply inflation itself; (2) the steady-state value of the optimal inflation target is positive for plausible model parameterizations; and (3) the optimal inflation target fluctuates following shocks to cohort-productivity growth and experience-accumulation growth.

Nevertheless, a modified form of "divine coincidence" continues to hold in response to all productivity disturbances: the output gap and the inflation *gap*, i.e., the gap between inflation and the optimal inflation target, can be closed simultaneously. Yet, this no longer implies a time-invariant inflation rate, unlike in models without supply-side turnover: positive shocks to the growth rate of cohort productivity (ii) decrease the inflation target and thus inflation under optimal policy, while positive shocks to experience productivity growth (iii) increase it. As in models without supply-side turnover, shocks to the growth rate of common productivity (i) leave the inflation target unaffected.

The optimal inflation target responds to shocks (ii) and (iii) because these shocks affect *the relative productivity* of new versus incumbent firms, unlike the common productivity shocks (i). Shocks (ii) and (iii) thus require adjustments in relative prices between new and existing firms to ensure efficiency. Since the prices of incumbent firms are typically sticky, it is optimal that the necessary relative price adjustments are brought about by new firms, whose prices are newly set upon entry and thus flexible. Therefore, shocks that cause the productivity of new firms to fall (rise) relative to incumbent firms make it optimal that new firms set higher (lower) prices, leading to an inflation target that is optimally higher (lower) than otherwise.

Despite supply shocks (ii) and (iii) moving the inflation target in potentially persistent ways, monetary policy should "look through" these shocks and keep the nominal rate unchanged. This is optimal following transitory or persistent shocks to cohort productivity growth (ii) and transitory shocks to experience productivity growth (iii).[1] Movements in expected inflation alone then cause the real interest rate to track its natural rate, so that changes in the nominal rate are not required.

Supply shocks can move the optimal inflation target, but naturally also affect the

[1]It is not true following persistent shocks to experience productivity growth, as we explain in the main text. It is also not true following common productivity shocks (i).

dynamics of efficient output and thus the natural rate of interest. We show that this creates new challenges for the correct identification of demand shocks in a setting with supply-side turnover. Specifically, a purely temporary positive shock to the growth rate of experience productivity (iii) causes output and inflation to both persistently increase under optimal monetary policy (which keeps nominal rates unchanged). This is similar to a standard demand shock, which also increases output and inflation in the absence of a nominal interest rate response. Traditional sign-based identification approaches for demand shocks are therefore prone to misidentify supply shocks as demand shocks in the presence of supply-side turnover. We show that the very same supply shock can also cause difficulties for high-frequency-based identification approaches of monetary policy shocks that rely on the stock market response to filter out so-called information effects that arise from news about shocks other than monetary policy shocks (Jarocinsky and Karadi (2020)).

The finding that supply shocks mimic demand shocks in a setup with firm turnover may also explain why measured aggregate productivity responds positively to an identified monetary policy loosening in the data, as documented in Christiano, Eichenbaum and Evans (2005): empirically identified demand shocks could be partially contaminated by supply shocks. This explanation contrasts with complementary explanations put forward in Meier and Reinelt (2024) and Baqaee, Farhi and Sangani (2024), who also consider a setup with heterogeneous firms and sticky prices. They emphasize that demand expansions lead to a compression of the cross-sectional mark-up distribution and thereby increase aggregate productivity. They do not analyze optimal monetary policy design and abstract from firm turnover.

More generally, the model with firm turnover gives rise to an alternative theory of demand shocks that is grounded in productivity shocks, akin to the imperfect information model presented in Lorenzoni (2009), where demand-like shocks arise from information noise about aggregate productivity; the work of Bai, Ríos-Rull and Storesletten (2025), where supply shocks generate demand-like responses in a setting with search frictions; and the work of Cesa-Bianchi and Ferrero (2021), in which sectoral supply shocks can generate aggregate effects that look like demand shocks.

The present analysis extends the analysis in Adam and Weber (2019), which focuses exclusively on the implications of firm turnover for the optimal inflation rate in a fully nonlinear setting. The present paper provides a linear-quadratic formulation of the optimal monetary policy problem, in particular a generalized welfare objective, a generalized New Keynesian Phillips curve, and a generalized expression for the natural rate of interest. The present analysis also connects directly to a large body of applied work that relies on linearized structural equations. In fact, we show that incorporating supply-side turnover requires only minimal changes to the widely-used linearized structural equations representing the standard New-Keynesian model. Nevertheless, these changes have strong implications for optimal monetary policy design.

The chapter is also related to the sticky-price literature that considers models with endogenous firm entry, e.g., Bilbiie, Fujiwara and Ghironi (2014), and models with endogenous firm entry and exit, e.g., Oikawa and Ueda (2018). They analyze how the steady-state inflation rate affects the entry and exit margins of firms and can improve welfare when these margins are inefficient. The present chapter takes the entry and exit margins as given, but goes beyond the analysis of optimal steady-state inflation.

The remainder of the chapter is structured as follows. Section 2 presents the sticky-price model with firm turnover, and section 3 explains how firm-level productivity dynamics affect the welfare-optimal inflation target. Section 4 presents the linear-quadratic optimal policy problem for the case with firm turnover and the dynamics for optimal inflation and output. Section 5 shows that it is optimal for monetary policy to look through certain supply disturbances. Section 6 explains how supply disturbances can generate very similar outcomes to demand disturbances and why this can lead to misidentification of demand shocks.

2 Economic Model

We consider a cashless economy with Calvo price rigidities and a firm life cycle that features firm turnover, as in Adam and Weber (2019). Firms enter the economy with a cohort-specific initial productivity level, accumulate technological experience during their lifetime, and randomly exit the economy. Unlike the standard New Keynesian model, the present setup gives rise to a setting in which the average productivity of price-adjusting firms is not necessarily equal to that of non-adjusting firms. As a result, the optimal steady-state inflation target generically differs from zero, and optimal inflation responds to supply-side disturbances that affect the relative productivity between new and incumbent firms. The monetary policy implications of these supply-side disturbances become particularly transparent with the linear Phillips curve and the quadratic welfare approximation that we derive in this section.

2.1 Household Problem

We follow Christiano, Trabandt and Walentin (2010) and consider a representative household with balanced-growth consistent preferences that are separable between consumption C_t and hours worked L_t:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\ln C_t - \frac{L_t^{1+\psi}}{1+\psi} \right), \tag{1}$$

where $\beta \in (0,1)$ denotes the discount factor, $\psi > 0$ the inverse Frisch elasticity of labor supply, and ξ_t a preference shock that evolves according to

$$\xi_t / \xi_{t-1} = e^{\widehat{\xi}_t},$$

with $\widehat{\xi}_t$ denoting a serially correlated shock such that $E[e^{\widehat{\xi}_t}] = 1$ and $E[\xi_t] = 1$. The household faces the flow budget constraint

$$C_t + \frac{B_t}{P_t} = \frac{W_t}{P_t} L_t + \int_0^1 \frac{\Theta_{jt}}{P_t} \, \mathrm{dj} + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) - T_t,$$

where B_t denotes nominal government bond holdings, P_t the aggregate price index, W_t the nominal wage rate, Θ_{jt} nominal profits from ownership of firm j, i_{t-1} the nominal interest rate, and T_t real lump sum taxes.

2.2 Supply Disturbances and Nominal Rigidities

In each period $t = 0, 1, \ldots$ there is a unit mass of monopolistically competitive firms indexed by $j \in [0, 1]$. Firm j produces output Y_{jt}, which enters as input into the production of the aggregate consumption good Y_t according to

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} \mathrm{dj} \right)^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $1 < \theta < \infty$ denotes the price elasticity of product demand. Let P_{jt} denote the price charged by firm j in period t. Firms can adjust prices with probability $1 - \alpha$ each period $(0 \leq \alpha < 1)$. The arrival of such a Calvo price adjustment opportunity is idiosyncratic and independent of all other exogenous random variables in the economy.

We augment this standard setting by a second price adjustment opportunity that arrives with idiosyncratic probability $\delta > 0$ each period and that is independent of the Calvo adjustment opportunities. This second adjustment opportunity, which we refer to as "δ-shock", arrives in conjunction with a firm-level productivity change, as described further below. Let $\delta_{jt} \in \{0, 1\}$ denote the idiosyncratic i.i.d. random variable governing this second price and productivity adjustment and let $\delta_{jt} = 1$ indicate the arrival of a δ-shock. We will interpret $\delta_{jt} = 1$ as capturing a firm turnover event in which firm j becomes permanently unproductive, thus exits the economy, and is replaced by a newly entering firm. For simplicity, we assign the same firm index j to the new firm, even though it operates a different production technology.

Alternatively, one can interpret $\delta_{jt} = 1$ as a product turnover event in which the demand for firm j's product disappears and where the same firm introduces a new product that is then produced with a different technology.

Letting L_{jt} denote the amount of labor used by firm j, firm output Y_{jt} is given by

$$Y_{jt} = A_t Z_{jt} L_{jt}, \tag{3}$$

where A_t captures the common productivity across all firms and Z_{jt} firm-specific productivity. Atkeson and Kehoe (2005) refer to Z_{jt} as the 'organization capital' of firm j.

Common productivity evolves according to

$$A_t = a_t A_{t-1},$$

where a_t is the (gross) growth rate of the common productivity in t. Firm-specific productivity evolves according to

$$Z_{jt} = \begin{cases} g_t Z_{jt-1} & \text{if } \delta_{jt} = 0 \\ Q_t & \text{if } \delta_{jt} = 1, \end{cases} \tag{4}$$

where g_t denotes the (gross) growth rate at which firms accumulate experience as they age, e.g., from learning-by-doing effects. The variable Q_t denotes the initial productivity level of the cohort of new firms that enters in period t.[2] We assume that

$$Q_t = q_t Q_{t-1}, \tag{5}$$

where q_t is the (gross) growth rate of the initial cohort-productivity level.

Each of the three different stochastic productivity trends in this setup, that is, (i) the common growth trend a_t, (ii) the cohort growth trend q_t, and (iii) the experience growth trend g_t, will have different implications for the optimal inflation target and the behavior of the natural rate of interest. We can decompose each of these trends into a mean component and fluctuations around the mean:

$$a_t = a \cdot e^{\widehat{a}_t}$$
$$q_t = q \cdot e^{\widehat{q}_t}$$
$$g_t = g \cdot e^{\widehat{g}_t},$$

where a, q, g denote mean components and $\widehat{a}_t, \widehat{q}_t, \widehat{g}_t$ are arbitrary serially correlated shocks whose unconditional means satisfy $E[e^{\widehat{a}_t}] = E[e^{\widehat{q}_t}] = E[e^{\widehat{g}_t}] = 1$. To obtain a well-defined

[2] Atkeson and Kehoe (2005) refer to the initial productivity endowment Q_t, which the firm is assigned when $\delta_{jt} = 1$, as the 'frontier of knowledge'.

steady state, we assume

$$(1-\delta)\left(g/q\right)^{\theta-1}<1. \tag{6}$$

Let s_{jt} denote the firm age, i.e., the number of periods that have elapsed since firm j last experienced a δ-shock.[3] Firm-specific productivity Z_{jt} in equation (4) is then given by

$$Z_{jt}=G_{jt}Q_{t-s_{jt}},$$

where

$$G_{jt}=\begin{cases} 1 & \text{for } s_{jt}=0 \\ g_t G_{jt-1} & \text{otherwise,} \end{cases}$$

and where Q_t follows equation (5). This shows that firm-specific productivity reflects the level of cohort productivity at the time of entry and the productivity gains from the accumulated experience since entry.

As usual, we define the aggregate price level as

$$P_t\equiv\left(\int_0^1 P_{jt}^{1-\theta}\mathrm{dj}\right)^{\frac{1}{1-\theta}}, \tag{7}$$

so that the gross inflation rate is given by

$$\Pi_t\equiv P_t/P_{t-1}.$$

Cost minimization in the production of final output Y_t implies

$$Y_{jt}=\left(P_{jt}/P_t\right)^{-\theta}Y_t. \tag{8}$$

Aggregation across firms yields the aggregate technology

$$Y_t=\frac{A_tQ_t}{\Delta_t}L_t, \tag{9}$$

[3] This means that $\delta_{j,t-s_{jt}}=1$ and that $\delta_{j,\widetilde{t}}=0$ for $\widetilde{t}=t-s_{jt}+1,...,t$.

where L_t denotes aggregate hours worked and

$$\Delta_t = \int_0^1 \left(\frac{Q_t}{G_{jt}Q_{t-s_{jt}}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta} \mathrm{dj} \tag{10}$$

the endogenous component to aggregate productivity, which depends on firms' relative prices.

2.3 Optimal Price Setting

Firms choose prices and hours worked to maximize expected discounted profits. Although price adjustment is subject to adjustment frictions, factor input can be chosen flexibly. Firms' sales are subject to a linear sales subsidy τ_t (sales tax if $\tau_t < 0$) and fluctuations in this subsidy (tax) induce time-variation in firms' desired mark-ups.[4]

Each period t, firms receive a δ-shock with probability δ and - conditional on not receiving a δ-shock - a Calvo shock with probability $1-\alpha$. If firm j receives either of the two shocks, it can freely choose a new optimizing price $P^\star_{jt}$, otherwise the firm's price is given by its previous price ($P_{jt} = P_{jt-1}$).[5]

Appendix A.1 shows that the optimal reset price $P^\star_{jt}$ is given by

$$\frac{P^\star_{jt}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right) = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t}, \tag{11}$$

where τ denotes the steady-state value of the sales subsidy and the aggregate variables N_t and D_t are independent of the firm index j. They are defined recursively as

$$N_t = \frac{W_t/P_t}{\Delta^e_t} + \alpha(1-\delta)\beta E_t\left[\left(\frac{\xi_{t+1}}{\xi_t}\right)(\Pi_{t+1})^{\theta}\left(\frac{q_{t+1}}{g_{t+1}}\right)N_{t+1}\right] \tag{12}$$

$$D_t = \frac{1+\tau_t}{1+\tau} + \alpha(1-\delta)\beta E_t\left[\left(\frac{\xi_{t+1}}{\xi_t}\right)(\Pi_{t+1})^{\theta-1}D_{t+1}\right], \tag{13}$$

[4] Aggregation of the heterogeneous firm model simplifies if desired mark-ups fluctuate due to variation in sales subsidies/taxes rather than due to variation in the elasticity of product demand.

[5] The case where non-reoptimized prices are indexed to a measure of lagged inflation will be considered in appendix A.

where Δ_t^e denotes the value of Δ_t when relative prices are efficient, which is given by

$$\Delta_t^e = \left(\int_0^1 \left(\frac{Q_t}{G_{jt} Q_{t-s_{jt}}} \right)^{1-\theta} dj \right)^{\frac{1}{1-\theta}},$$

and which evolves recursively according to

$$\left(\Delta_t^e\right)^{1-\theta} = \delta + (1-\delta) \left(\Delta_{t-1}^e q_t / g_t \right)^{1-\theta}.$$

Equation (11) shows that the optimal relative reset price of a firm depends on aggregate variables and the ratio between its own productivity ($A_t Q_{t-s_{jt}} G_{jt}$) and the productivity of a new firm in period t ($A_t Q_t$). In the standard New Keynesian setting with homogeneous firm, this productivity ratio is always equal to one, but this will generally not be true here. Only in the special case where $g_t = q_t$ for all t, the present setting reduces to one with homogeneous firm productivity.[6] One then has $\Delta_t^e = 1$, so that equations (11)-(13) reduce to the ones in the textbook New Keynesian model.

As is standard in the New Keynesian literature, we assume that the steady-state sales subsidy τ eliminates the steady-state distortions arising from monopolistic competition:

$$\frac{\theta}{\theta-1} \frac{1}{1+\tau} = 1. \tag{14}$$

The sales subsidy τ_t is allowed to fluctuate around this steady-state level and will give rise to mark-up disturbances in the Phillips curve.

2.4 Government

To close the model, we consider a government which faces the budget constraint

$$\frac{B_t}{P_t} = \frac{B_{t-1}}{P_t}(1 + i_{t-1}) + \tau_t \int_0^1 \left(\frac{P_{jt}}{P_t} \right) Y_{jt} \text{ dj} - T_t.$$

[6] This assumes that the initial distribution of productivities is also degenerate, otherwise this will hold only asymptotically.

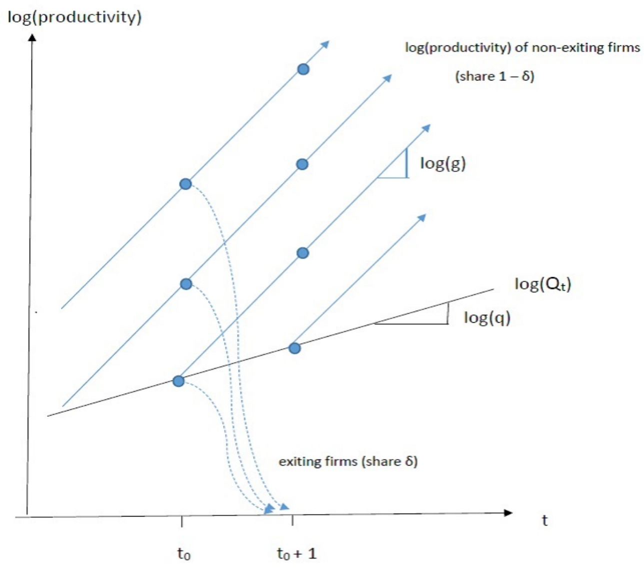

Figure 1: Productivity dynamics in a setting with firm entry and exit

The government levies lump sum taxes T_t, so as to implement a bounded state-contingent path for government debt B_t/P_t.[7] Since we consider a cashless limit economy, there are no seigniorage revenues, even though the central bank controls the nominal interest rate. We furthermore assume that monetary policy is not constrained by a lower bound on nominal interest rates.

3 Firm-Level Productivity Dynamics and Optimal Inflation

This section illustrates the firm-level productivity dynamics implied by our setup and the crucial role played by inflation in achieving efficiency of relative prices.[8] Specifically, it shows that the optimal inflation rate is related to the relative productivity between new and incumbent firms.

Figure 1 illustrates the firm-level productivity dynamics for the empirically plausible case in which the cohort trend exceeds unity ($q > 1$), but is less strong than the experience

[7] The household's transversality condition will then automatically be satisfied in equilibrium.

[8] To insure that the chapter is self-contained, this section reviews some of the key insights from Adam and Weber (2019).

trend ($g > q$). To simplify the exposition, the figure depicts the deterministic productivity dynamics only and abstracts from the common productivity trend a, which does not affect the relative productivity between firms.

The line labeled $\log(Q_t)$ in figure 1 indicates the cohort trend and captures the productivity of newly entering firms at each point in time. Lines departing from the cohort trend capture the productivity dynamics of the entering cohorts over time. Since $g > q$, the productivity of incumbent firms grows faster than the productivity of new entrants, so that incumbent firms are more productive and thus larger than newly entering firms. In experience-adjusted terms, however, newly entering firms are the most productive firms in the economy. The dashed arrows indicate the productivity losses of exiting firms that have been hit by a δ-shock. The entry and exit dynamics imply an exponential distribution for firm age.[9]

Importantly, once firms have entered the economy, their productivity relative to other incumbent firms remains unchanged: all incumbent firms sustain the same experience trend (g) and the same common productivity trend (a). (The latter is not shown in the figure.) This holds true even for the case with productivity shocks. Therefore, if the initial relative prices of new firms reflect (inversely) their relative productivity, efficiency cannot be improved further by adjusting prices of incumbent firms in response to productivity disturbances.[10] To obtain efficiency of *all* relative prices, it is thus sufficient that newly entering firms set a relative price that (inversely) reflects their productivity relative to that of the average incumbent firm. Since newly entering firms in figure 1 have lower productivity than the average incumbent firm, the prices of newly entering firms in figure 1 should optimally be higher than the price of the average incumbent firm, causing inflation to be optimal.[11]

[9] Coad (2010) shows that such an age distribution is empirically plausible and how it generates, together with (productivity) growth shocks, a Pareto distribution for firm size, in line with the observed firm size distribution.

[10] This is not true following mark-up disturbances as we discuss later on.

[11] The average price of exiting firms in the period before being hit by the exit shock is equal to the average price of incumbent firms, so that the prices of exiting firms are replaced by the higher prices of newly entering firms. This is consistent with empirical evidence in Bils (2009) who shows that a large part of inflation is due to the higher prices charged for new products.

4 The Optimal Monetary Policy Problem

Despite the presence of heterogeneous firms with different accumulated experience and different initial cohort productivity, it is possible to aggregate output across firms in closed form and to derive a linear-quadratic approximation of the policymaker's Ramsey problem. The approximate policy problem features a generalized quadratic objective function and a generalized linear New Keynesian Phillips Curve. The approximate problem transparently reveals how the stochastic productivity trends (a_t, g_t, q_t), the tax/subsidy shocks (τ_t) and the preference shocks (ξ_t) shape the monetary policy trade-offs in the presence of firm turnover and firm heterogeneity. Since productivity trends also affect the steady-state values of the inflation target and the natural rate of interest, we start by considering the effects of these trends on the efficient deterministic steady-state outcomes. Subsequently, we present the linear-quadratic stabilization problem around this steady state, taking also into account the economic shocks.

4.1 Optimal Steady-State Inflation and Real Rate

The optimal steady-state (gross) inflation rate, which is equal to the steady-state inflation target, is equal to[12]

$$\Pi^{\star} = \frac{g}{q},$$

where g denotes the (gross) experience productivity trend and q the (gross) cohort productivity trend. This is a special case of the multi-sector steady-state result derived in Adam and Weber (2023), see also Adam and Weber (2025) for a literature overview.

The optimal gross inflation rate is larger than one whenever $g > q$. Incumbent firms then accumulate experience productivity at a rate faster than the rate at which successively entering cohorts become more productive. As a result, young firms are less productive than incumbent firms, in line with empirical evidence, e.g., Adam, Renkin and Züllig (2025). Similarly, when the model is interpreted as one featuring product turnover instead of firm turnover, the empirical evidence in Adam and Weber (2023) for the U.K.

[12] As in the standard model, the optimal steady-state inflation rate is defined as the long-run outcome of the Ramsey plan in the absence of economic shocks.

and Adam, Gautier, Santoro, and Weber (2022) for Germany, France, and Italy shows that the relative price of products falls over the product life cycle. This also requires $g > q$.

Interestingly, the optimal steady-state inflation target is independent of the turnover rate δ. Although a higher δ implies that more relatively unproductive firms that set a high relative price are entering, it also implies that the average incumbent firm has had less time to accumulate experience, so the relative price does not need to be as high as with a lower δ. As it turns out, the two effects exactly offset each other. As a result, the optimal steady-state inflation rate is independent of δ.

The real steady-state (gross) interest rate in the efficient equilibrium, R^e, is equal to

$$R^e = \frac{aq}{\beta}.$$

It decreases with the discount factor β and increases with the steady-state output and the aggregate productivity growth rate aq.[13] With a positive steady-state growth rate of the economy, $aq > 1$, the real (gross) interest rate satisfies $R^e > 1$. Moreover, the nominal (gross) steady-state interest rate is larger than one if

$$\frac{ag}{\beta} > 1. \tag{15}$$

We can then abstract from the lower-bound constraint on nominal interest rates in our linear-quadratic approximation for sufficiently small economic disturbances.

4.2 The Linear-Quadratic Policy Problem

To approximate the policymaker's Ramsey problem, we impose the following assumptions:

Assumption 1 1. Initial prices in $t = -1$ inversely reflect firms' productivity, to a

[13] The trend of experience productivity g generates only level effects for aggregate productivity because accumulated experience is ultimately lost when firms leave the economy.

first-order approximation:

$$P_{j,-1} = \Phi \frac{1}{Q_{-1-s_{j,-1}} G_{j,-1}} + O(2) \qquad \text{for all } j \in [0,1] \text{ and some } \Phi > 0,$$

where $O(2)$ is a second-order approximation error.

2. The steady-state sales subsidy is efficient, i.e., equation (14) holds.

3. The nominal (gross) steady-state interest rate is larger than one, i.e., inequality (15) holds.

These assumptions are all standard in the sticky-price literature: the first insures that inefficient price dispersion is of second order in the initial period; the second assumption guarantees that the sticky-price steady state is efficient, so that it is sufficient to approximate the nonlinear Phillips curve and other equilibrium conditions to first order; the third assumption insures that the zero lower-bound constraint on nominal rates does not bind in the steady state.[14]

We then approximate the optimal policy problem around the efficient steady state and define the log-deviation of inflation and of the inflation target from the (nonzero) steady-state value as:

$$\pi_t \equiv \ln \Pi_t - \ln \Pi^\star, \tag{16}$$

$$\pi_t^\star \equiv \ln \Pi_t^\star - \ln \Pi^\star, \tag{17}$$

where

$$\Pi_t^* = \left(\frac{1 - \delta (\Delta_t^e)^{\theta - 1}}{1 - \delta} \right)^{\frac{1}{\theta - 1}}. \tag{18}$$

The output gap is defined as

$$x_t \equiv \ln Y_t - \ln Y_t^e, \tag{19}$$

[14]Unlike in the canonical model without economic growth, this does not follow from time discounting alone in the present setting.

where Y_t^e denotes the efficient level of output, which is given by

$$Y_t^e = \frac{A_t Q_t}{\Delta_t^e}.$$

We define the log-deviation of the natural rate of interest from its steady-state value as

$$r_t^n \equiv \ln R_t^e - \ln R^e, \tag{20}$$

and the log-deviation of the nominal interest rate from its steady-state value as

$$\hat{\imath}_t \equiv \ln(1 + i_t) - \ln(ag/\beta). \tag{21}$$

The mark-up shock, which captures the effect of time variation in the sales subsidy, is defined as

$$u_t \equiv \frac{\kappa}{1+\psi}\left(\ln\frac{1}{1+\tau_t} - \ln\frac{1}{1+\tau}\right), \tag{22}$$

where

$$\kappa \equiv \frac{(1-\alpha\rho)(1-\alpha\beta\rho)}{\alpha\rho}(1+\psi), \tag{23}$$

and

$$\rho \equiv (1-\delta)(\Pi^\star)^{(\theta-1)} < 1. \tag{24}$$

Using these definitions, we can state our main result:

Proposition 1 *Suppose Assumption 1 holds. A linear-quadratic approximation to the* *optimal monetary policy problem is then given by*

$$\max_{\{x_t,\pi_t,\hat{\imath}_t\}_{t=0}^{\infty}} - E_0 \sum_{t=0}^{\infty} \beta^t \left((\pi_t - \pi_t^{\star})^2 + \lambda x_t^2 \right) \tag{25}$$

$$s.t.:$$

$$\pi_t - \pi_t^{\star} = \beta E_t \left[\pi_{t+1} - \pi_{t+1}^{\star} \right] + \kappa x_t + u_t \tag{26}$$

$$x_t = E_t x_{t+1} - (\hat{\imath}_t - E_t \pi_{t+1}) + r_t^n, \tag{27}$$

with $\lambda \equiv \kappa/\theta$ *and*

$$\pi_t^{\star} = \rho \pi_{t-1}^{\star} + (1-\rho) \cdot (\widehat{g}_t - \widehat{q}_t), \tag{28}$$

where $\pi_{-1}^{\star}$ *is given. The natural rate evolves according to*

$$r_t^n = E_t \left[\widehat{a}_{t+1} + \widehat{g}_{t+1} - \widehat{\xi}_{t+1} - \pi_{t+1}^{\star} \right]. \tag{29}$$

The proof of the proposition can be found in appendices A, B and C, which consider a generalized setup that also allows for price indexation during periods in which prices are not optimally reset. The policy problem in Proposition 1 differs from the one in the canonical model without firm turnover in a number of important ways:

1. The welfare objective (25) depends on the gap between actual inflation and a time-varying optimal inflation target π_t^*. In the canonical homogeneous firm model, the optimal inflation rate is constant.

2. The same is true for the New Keynesian Phillips curve (26). With firm turnover, it contains two new terms: the time varying-inflation target and the expectations of this target in the next period. The Phillips curve can thus alternatively be expressed as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t + u_t^{\star}, \tag{30}$$

where $u_t^* \equiv \pi_t^{\star} - \beta E_t \pi_{t+1}^{\star}$. Viewed through this lens, there are now two different shocks present in the Phillips curve: (i) traditional mark-up shocks u_t, and (ii) the new shocks $u_t^\star$ that capture time variation in the inflation target. Equation (28) shows that purely *temporary* shocks to experience or cohort productivity growth ($\widehat{g}_t$, $\widehat{q}_t$) give rise to *persistent* disturbances $u_t^\star$ in the Phillips Curve. The model with firm turnover thus delivers an alternative theory of persistent Phillips curve disturbances that is grounded in productivity movements. This is of interest because persistent Phillips curve disturbances have been found to be quantitatively important when bringing the New Keynesian Phillips Curve to the data (Smets and Wouters (2003, 2007)). At the same time, the traditional micro-foundations for these shocks, i.e., variations in taxes, subsidies, or firm monopoly power, have been called into question, e.g., Chari, Kehoe and McGratten (2009).

Clearly, if the shocks $\widehat{g}_t$ and $\widehat{q}_t$ follow a moving average process in which the initial impact is reversed in the subsequent period with a factor $-\rho$, then $u_t^\star$ becomes a purely temporary disturbance.

3. The policy problem (25)-(27) is (generically) approximated around a steady state with nonzero optimal inflation ($\ln \Pi^\star = \ln g/q \lesseqgtr 0$). Under plausible model calibrations, the steady-state inflation rate is positive; see the discussion in section 4.1. This brings the normative predictions of the model in line with the practice of central banks, who tend to target positive inflation rates. In contrast, the optimal inflation target in the canonical homogeneous firm model is zero.

4. In the canonical homogeneous firm model, only mark-up shocks move optimal inflation. In the presence of firm turnover, also productivity disturbances move optimal inflation, because the optimal inflation target in equation (28) depends on the shocks to experience productivity growth $\widehat{g}_t$ and cohort productivity growth $\widehat{q}_t$. The dynamics of the optimal inflation target will be explored in detail in the next section.

5. The slope of the Phillips κ in equation (23) generally differs from that in the standard model and depends on the steady-state inflation rate via the parameter ρ. For the special case in which the optimal steady-state inflation rate is given by $\ln \Pi^\star =$

$\ln g/q = 0$, the slope of the Phillips curve is

$$\kappa|_{\Pi^\star=1} = \frac{(1-\alpha(1-\delta))\,(1-\alpha(1-\delta)\beta)}{\alpha(1-\delta)}(1+\psi), \tag{31}$$

where $\alpha(1-\delta)$ is the overall degree of price rigidity, taking into account that newly entering firms can also freely set their price. The slope of the Phillips curve is then identical to that in the canonical homogeneous firm model with overall price rigidity $\alpha(1-\delta)$. However, in the present setup, the slope of the Phillips curve in equation (23) falls with the optimal steady-state inflation rate $\Pi^\star$ because

$$\frac{\partial\kappa}{\partial\Pi^\star} < 0$$

for all $\Pi^\star$ satisfying the existence condition (6). The Phillips curve is therefore flatter than in the canonical New Keynesian model, whenever the optimal steady-state inflation rate is greater than zero ($\ln\Pi^\star > 0$). A positive optimal inflation target implies that firm's relative price decreases over time, so that future periods are associated with larger demand than nearby periods. Firms' horizon thus effectively lengthens because firms care more about profits in future periods (Blanco et al. (2024)). As a result, firms react less strongly to current demand fluctuations compared to a situation in which optimal inflation is zero. The effect is similar to that arising when linearizing the standard model around a (non-optimal) positive inflation target, see equation (20) in Ascari (2004), but differs slightly because the present model is linearized around an optimal positive steady-state inflation rate.

6. The natural rate of interest r_t^n is also affected by the new productivity disturbances $(\widehat{g}_t, \widehat{q}_t)$, see equations (28) and (29). The induced natural rate dynamics will be discussed in detail in the next section.

For the special case in which $g_t = q_t$ for all t, the setup in Proposition 1 reduces to one with homogeneous firm productivity. The proposition then captures the case of the canonical New Keynesian model in which the optimal steady-state inflation is zero, the inflation target is not time-varying (see equation (28)), and the Phillips curve has its standard slope (31).

Even when $g_t \neq q_t$, the monetary policy problem (25)-(27) remains closely related to the problem in the canonical New Keynesian model with homogeneous firms (Woodford (2011)). Specifically, in the absence of a binding lower-bound constraint, the Euler equation (27) does not constrain the problem and can be eliminated from the optimization problem, together with the interest rate $\hat{\imath}_t$. The resulting simpler optimal policy problem, which involves only the output gap (x_t) and the inflation gap ($\pi_t - \pi_t^\star$), is then the same as in the canonical model if one replaces the inflation gap ($\pi_t - \pi_t^\star$) by inflation (π_t) and adjusts the slope of the Phillips curve accordingly. Therefore, all lessons from the canonical homogeneous firm model about the optimal behavior of the output gap and inflation apply to the output gap and the inflation *gap* in the model with firm turnover. Fundamentally new insights for optimal stabilization policy thus only arise for the shocks $(\widehat{q}_t, \widehat{g}_t)$ that move the optimal inflation target.

The next sections explore in greater detail how these shocks affect the dynamics of the inflation target, the natural rate, and efficient output growth.

4.3 Dynamics of the Inflation Target and Natural Rate

This section discusses how the inflation target and the natural rate of interest respond to shocks to experience productivity growth ($\widehat{g}_t$) and shocks to cohort productivity growth ($\widehat{q}_t$). We focus on these shocks because the inflation target does not respond to other shocks and because the response of the natural rate to other shocks is the same as in the canonical model without turnover.

Equation (28) reveals that the response of the optimal inflation target to shocks depends exclusively on the parameter ρ. According to equation (24), the parameter ρ is approximately equal to one minus the turnover rate δ, when the optimal gross steady-state inflation target Π^* is close to one. A lower turnover rate thus implies that the inflation target responds initially less strongly to disturbances but then more persistently so.

Turnover rates differ depending on whether one interprets the model as one featuring firm turnover or product turnover, but overall turnover rates tend to be low. Therefore, the model-implied optimal inflation target will display rather persistent fluctuations over time. In particular, the firm exit rate in the U.S. is around 7.5% per year (Crane et al. (2022)). For a quarterly model, this implies $\rho \approx 1 - \delta = (1 - 0.075)^{(1/4)} \approx 0.98$ and thus a highly persistent inflation target. When also temporary firm closures are included, the exit rate increases to about 10% per year, which implies $\rho \approx 0.974$, or a half-life of shocks of about 26 quarters.

The turnover rate for products tends to be much higher than that for firms. Using the product definition of Adam and Weber (2019), it is around 6% per month for the U.K. economy.[15] For a quarterly model, this implies $\rho \approx 1 - \delta = (1 - 0.06)^3 \approx 0.83$ or a half-life of shocks of around 4 quarters.

Given this discrepancy, we illustrate the results below using an intermediate persistence value of

$$\rho = 0.95,$$

which implies a half-life of shocks of about 14 quarters, which is close to the average of the half-lives implied by firm turnover and product turnover.

The following section considers the effects of shocks to experience growth; we then consider shocks to cohort productivity growth.

4.3.1 Temporary Experience Accumulation Growth Shocks

The upper row of figure 2 displays the impulse response of the optimal inflation target (left panel) and the natural rate (right panel) to a one time, one percent increase in experience productivity growth ($\widehat{g}_0 = 0.01$) in $t = 0$.[16] It shows that such a shock leads to a small but very persistent increase in the optimal inflation target and a small and very persistent decrease in the natural rate.

The natural rate response is easy to understand: temporarily accelerated experience

[15] This number is obtained by adding comparable and non-comparable substitutions within product identifiers in the CPI, see their table 2.

[16] As discussed before, the figure assumes $\rho = 0.95$

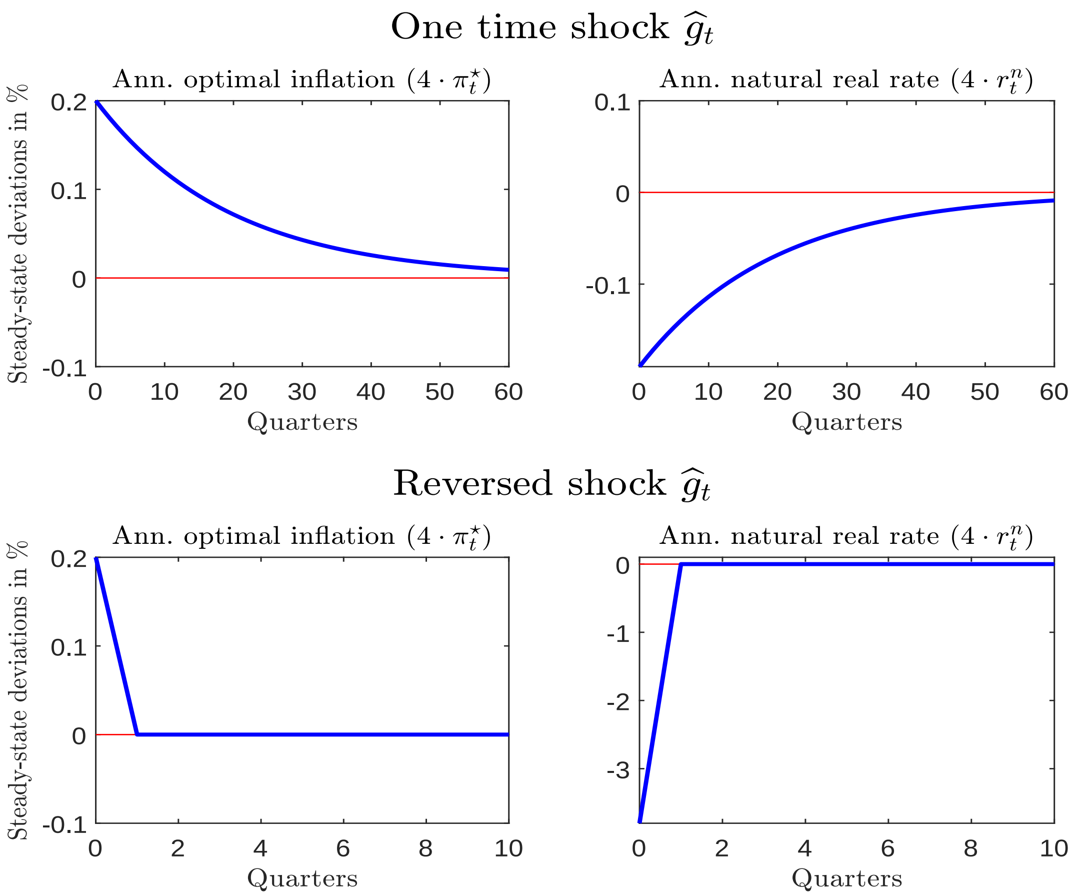

Figure 2: Inflation target and natural rate response to temporary $\widehat{g}_t$ shocks.

accumulation makes existing firms more productive, and thus leads to a jump in output. Due to firm exit, accumulated experience is lost over time, so that output slowly reverts to its steady state value. The declining output profile implies a lower equilibrium real rate.

To understand why the inflation target persistently increases, note that the shock to the growth rate of experience productivity $\widehat{g}_t$ makes incumbent firms more productive, while leaving the productivity of newly entering firms unaffected, also in future periods. Efficiency then requires that the price charged by new firms increases relative to the price charged by incumbent firms. Since incumbent firms' prices are partially sticky, it is optimal to implement the required relative price change via an upward adjustment in the nominal price charged by new firms, which raises the price level in each period.[17] Since the productivity increase for incumbent firms disappears only slowly via the gradual exit

[17] Having incumbent firms reduce their nominal prices is inefficient as price stickiness prevents all incumbent firms from doing so at the same time, causing inefficient price dispersion.

of incumbent firms, the increase in the inflation target is rather persistent.

The lower panel of figure 2 depicts the impulse response to a one percent increase of experience productivity growth in $t = 0$ that is followed by productivity growth of $-\rho$ percent in the subsequent period. This (partial and fully anticipated) reversal causes the effect of the initial shock on all variables to be purely temporary.[18] The figure shows that the impact effect on the inflation target is then the same as in a setting without a shock reversal. The real rate response, however, is much stronger because output now reverts to its pre-shock value after just one period.

4.3.2 Temporary Cohort Productivity Growth Shocks

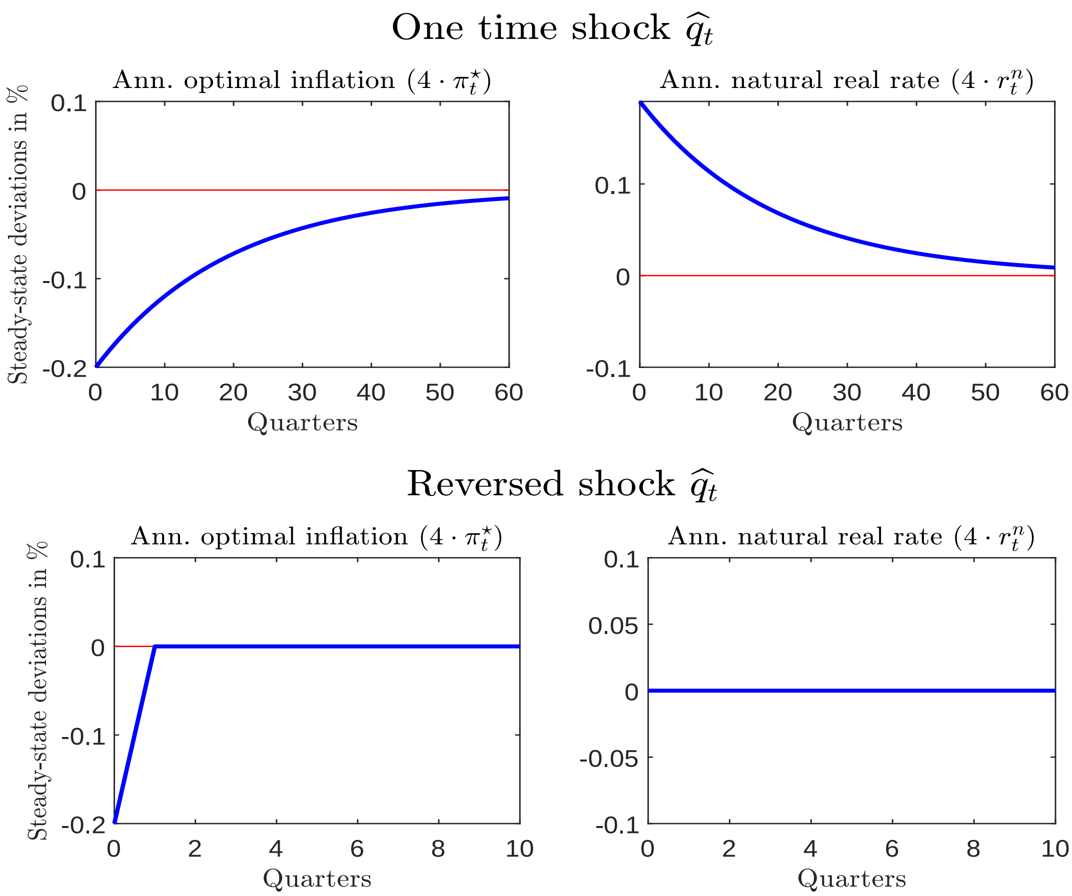

Figure 3: Inflation target and natural rate response to temporary $\widehat{q}_t$ shocks.

The upper row of figure 3 displays the impulse response of the optimal inflation target (left panel) and the natural rate (right panel) to a one time, one percent increase in cohort

[18] A full reversal is not required because part of the initial positive experience growth is automatically lost in the subsequent period due to firm exit.

productivity growth ($\widehat{q}_0 = 0.01$) in $t = 0$. It shows that the optimal inflation target falls persistently and the natural rate of interest increases persistently.

Again, the increase in the natural rate is easy to understand. Since new firms become permanently more productive, the entry dynamics generate a slow and permanent increase in the level of output. The increasing profile in output is associated with an increase in the natural rate.

To understand the response of the inflation target, note that new firms become permanently more productive, while the productivity of incumbent firms is now unaffected. Efficiency then requires that the price charged by new firms falls relative to that charged by incumbent firms. Again, it is efficient to implement this relative price adjustment via new firms, as their prices are flexible. Since new firms should charge lower prices, the optimal inflation target falls. As before, this effect is rather persistent and only fades as the new (and more productive) firm cohorts gradually replace the older firm cohorts.

The lower panel of figure 3 depicts the impulse response to a one percent increase in cohort productivity growth in $t = 0$ that is followed by cohort productivity growth of $-\rho$ percent in the subsequent period. This (fully anticipated) reversal causes the effects on the inflation target and the natural rate to be temporary.[19] Figure 3 shows that this shock generates the same impact period response for the inflation target as the shock without a reversal, but leaves the natural rate unchanged. The latter occurs because the shock and its partial reversal move output to a permanently higher level, thus generate neither an increasing nor a decreasing profile for output after the impact period.[20]

4.3.3 Permanent Shocks to Experience and Cohort Productivity Growth

We now consider the effects of permanent movements of $\widehat{g}_t$ and $\widehat{q}_t$ on the optimal inflation target and the natural rate of interest. Such permanent movements may result from structural change in the economy that affects the rate of experience accumulation or the rate of cohort productivity growth.

[19]However, unlike in the case with a partially reversed experience growth shock, the effect on output is not temporary.

[20]It is not possible to construct a shock process for cohort productivity growth that moves inflation target, natural rate and output only in the impact period.

It follows from equation (28) that permanent changes in $\widehat{g}_t$ or $\widehat{q}_t$ transmit in the long run one for one to the inflation target, albeit with different signs. Yet, since ρ is close to one, the transition will be relatively slow.

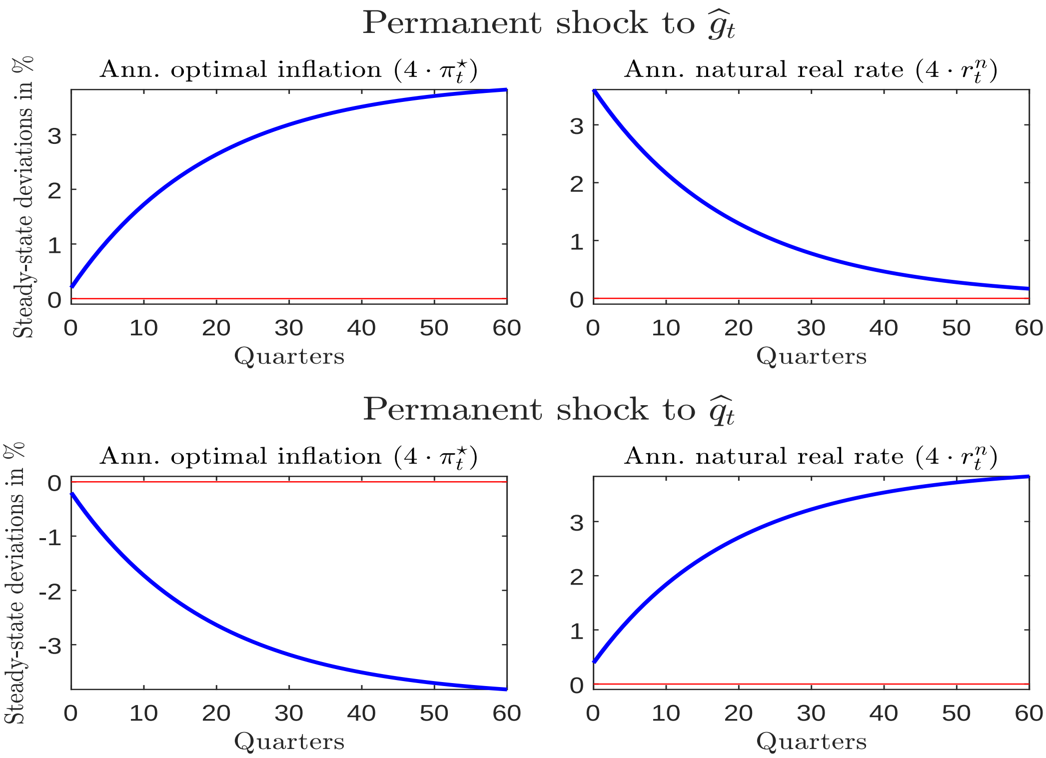

Figure 4: Inflation target and natural rate: permanent shocks to $\widehat{g}_t$ and $\widehat{q}_t$.

Figure 4 illustrates this fact. The top row depicts the impulse response of the optimal inflation target (left panel) and the real rate (right panel) to a permanent, one percent increase in $\widehat{g}_t$. The optimal inflation target gradually rises, ultimately increasing by four percent in annualized terms. The natural rate, however, shoots up strongly and only gradually decreases as the growth rate of output reverts to its pre-shock value: higher rates of experience accumulation only generate a level increase in output, as accumulated experience is ultimately lost due to firm turnover.

The bottom row in figure 4 depicts the impulse response of the optimal inflation target (left panel) and the real rate (right panel) to a permanent, one percent increase in $\widehat{q}_t$. The inflation target now gradually declines, ultimately decreasing by four percent in annualized terms. At the same time, economic growth gradually accelerates as more and more young firms with higher productivity enter the economy. As a result, the natural rate increases and reaches a permanently higher level, reflecting the higher growth rate of the economy.

4.4 The Dynamics of Efficient Output Growth

The efficient growth rate of real output is given by

$$\widehat{\gamma}_t^e = \rho\widehat{\gamma}_{t-1}^e + (\widehat{a}_t - \rho\widehat{a}_{t-1}) + \rho(\widehat{g}_t - \widehat{g}_{t-1}) + (1-\rho)\widehat{q}_t \ , \tag{32}$$

and depends on all three supply disturbances, see the derivations in appendix B.5.

A temporary increase in common productivity growth $\widehat{a}_t$ increases output growth once but has no additional effects on output growth in subsequent periods, so the economy is catapulted to a permanently higher output level.

A temporary increase in experience growth $\widehat{g}_t$ also increases output in the current period, with an effect that is close to one, due to the large share of incumbent firms in the economy. However, experience growth shocks result in negative output growth in subsequent periods because the long-term output effect of these shocks is zero: since no firm remains in the economy forever, increased experience will ultimately be lost again. Therefore, a temporary shock to experience productivity growth results in a persistent increase in the level of output that gradually fades out over time.

In contrast, a temporary increase in the growth rate of cohort productivity $\widehat{q}_t$ results in a permanent increase in long-run level of output. Yet, unlike with shocks to common productivity growth ($\widehat{a}_t$), the effect is initially small and gradually builds up over time, as only some firms enter the economy in each period.

Equation (32) also shows that permanent changes in common productivity growth $\widehat{a}_t$ increase output growth permanently and one for one. Permanent changes in cohort productivity growth $\widehat{q}_t$ also permanently increase output growth, but the effect again builds up only gradually over time. In contrast, permanent changes to experience productivity growth $\widehat{g}_t$ do not increase output growth in the long run, instead only leading to permanent changes in the level of output.

4.5 Optimal Stabilization Policy

This section discusses optimal monetary policy and its implementation via targeting and instrument rules, highlighting similarities and differences with the canonical New Keynesian model without supply-side turnover.

Under commitment, the first-order conditions to problem (25)-(27) imply the targeting criterion

$$\pi_t - \pi_t^\star = -\frac{\lambda}{\kappa}(x_t - x_{t-1}). \tag{33}$$

The targeting criterion is very similar to the one in the canonical New Keynesian model, but features the inflation gap instead of inflation on the left-hand side.

Commitment to the targeting criterion (33) implements the Ramsey outcome as the locally unique rational expectations equilibrium. In the Ramsey outcome, the output gap and the inflation gap can both be closed in response to the disturbances $(\widehat{a}_t, \widehat{q}_t, \widehat{g}_t, \widehat{\xi}_t)$, so that output follows its efficient level and inflation follows the optimal inflation target. In contrast, mark-up shocks u_t generate a trade-off between output gap and inflation *gap* stabilization and thus lead to a deviation of output from its efficient level and/or of inflation from its optimal target, as in the canonical model without turnover. Since mark-up shocks do not lead to movements in the inflation target $\pi_t^\star$, the optimal response to mark-up shocks is identical as in the canonical setup without turnover.

Defining the natural nominal interest rate as

$$\hat{\imath}_t^n \equiv r_t^n + E_t \pi_{t+1}^\star, \tag{34}$$

a Taylor rule of the form

$$\hat{\imath}_t = \hat{\imath}_t^n + \alpha_\pi \left(\pi_t - \pi_t^\star\right) \tag{35}$$

implements the Ramsey optimal monetary policy response to the disturbances $(\widehat{a}_t, \widehat{q}_t, \widehat{g}_t, \widehat{\xi}_t)$ as a locally unique equilibrium outcome, provided $\alpha_\pi > 1$.[21] The determinacy properties are thus identical to the ones in the canonical model without turnover and do not depend

[21] As in the canonical model without turnover, a Taylor rule does not implement the optimal policy response to mark-up disturbances u_t.

on the fact that the optimal steady-state inflation rate in the model with turnover generally differs from zero or the fact that the slope of the Phillips curve is generally different. However, for the rule (35) to be optimal in response to the shocks $(\widehat{a}_t, \widehat{q}_t, \widehat{g}_t, \widehat{\xi}_t)$, it must account for time-variation in the inflation target, both in the intercept and the inflation gap, unlike in the canonical model.

Interestingly, the determinacy conditions in the canonical model become more stringent when linearizing it around a positive inflation steady state, see Coibion and Gorodnichenko (2011) and Ascari and Sbordone (2014). This is not necessarily the case for the present model with turnover: empirically plausible model parameterizations ($g > q$) imply that the optimal steady-state inflation rate is positive, and linearizing the model with turnover around this optimal positive inflation rate leads to the standard determinacy condition $\alpha_\pi > 1$, as shown above.[22]

5 "Looking Through" Supply Disturbances

Monetary policymakers often express their desire to "look through" supply disturbances, see Powell (2023) and Lagarde (2025) for recent examples. However, this desire is hard to justify through the lens of a canonical New Keynesian model: under optimal monetary policy, common productivity disturbances move the natural real rate and output but do not move inflation. Nominal rates thus need to move to keep inflation unchanged, which is the opposite of a look-through approach. Supply side shocks that arise from movements in the desired markup also do not justify a 'look through' approach in the canonical model: they generate a policy trade-off between output gap and inflation stabilization and require an appropriate interest rate response under optimal policy.

These results continue to be true in the setup with firm turnover: neither disturbances to common productivity growth nor disturbances to desired mark-ups rationalize a look-through approach. However, the new supply disturbances $\widehat{g}_t$ and $\widehat{q}_t$ can justify a look-

[22]This is so because $\Delta_t = \Delta_t^e + O(2)$ when approximating either of the models around their respectively optimal steady-state inflation rates. The dynamics of Δ_t are then pinned down by the dynamics of Δ_t^e to first order and do not have endogenous first-order effects on the equilibrium. This is no longer the case when approximating around a suboptimal steady-state inflation rate.

through approach as being optimal. In particular, purely temporary shocks $\widehat{g}_t$ and purely temporary and persistent shocks $\widehat{q}_t$ do not move the natural nominal rate, which is given by:[23]

$$\widehat{\imath}_t^n = E_t \left[\widehat{a}_{t+1} + \widehat{g}_{t+1} - \widehat{\xi}_{t+1} \right]. \tag{36}$$

Following these disturbances, it is thus *not* optimal to adjust the policy rate, despite the fact that these disturbances generate persistent movements in output growth and the inflation rate, which follows the inflation target, see sections 4.3 and 4.4.

Figure 5 illustrates this outcome by depicting the Ramsey optimal response of inflation, detrended output and nominal rates.[24] The top row considers a positive and temporary experience productivity growth shock $\widehat{g}_0 > 0$ and the bottom row a positive and temporary cohort productivity growth shock $\widehat{q}_0 > 0$.[25] The figure shows that output and inflation respond persistently and in the same direction following experience growth shocks, but in opposite directions following cohort growth shocks. The optimal monetary policy response is to keep nominal rates unchanged.

The look-through approach continues to be optimal when shocks to cohort productivity growth $\widehat{q}_t$ are persistent. Expectations of future cohort productivity growth do not affect the natural nominal rate, see equation (36), because the effects of these shocks on the natural real rate and on the expected inflation target offset each other exactly.[26]

This implies that a period in which the initial productivity of young firms is persistently accelerating, as is generally considered to have happened during the New Economy boom in the mid-to-late 1990s in the United States, is a period in which aggregate growth accelerates, inflationary pressures become weaker, all the while nominal interest rates optimally stay constant. This finding broadly rationalizes Federal Reserve Chairman Alan Greenspan's response to the U.S. New Economy boom.

[23] This follows from equations (29) and (34).

[24] Detrended output is defined as $\widetilde{y}_t = \ln Y_t - \ln \overline{\Gamma}_t^e$, where $\overline{\Gamma}_t^e$ is the deterministic output level emerging in the absence of the shock, as derived in appendix B.5.

[25] As before, the figure assumes $\rho = 0.95$ and considers shocks of one percent. No other parameter matters for the impulse response dynamics in figure 5.

[26] This is not true for persistent experience growth shocks $\widehat{g}_t$. Following such shocks, monetary policy must optimally increase nominal interest rates, see equation (36).

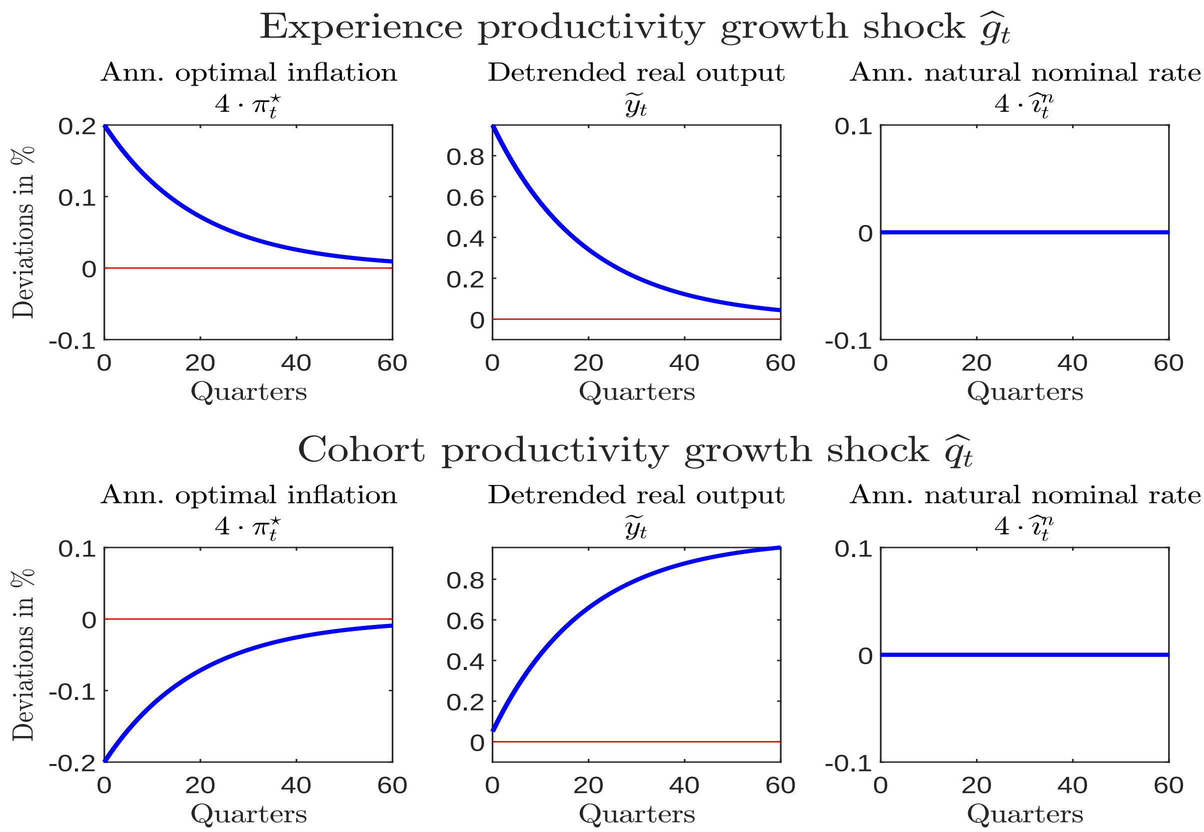

Figure 5: Looking through supply shocks: Impulse responses of temporary shocks to $\widehat{g}_t$ and $\widehat{q}_t$.

6 Supply Shocks Mimic Demand Shocks

This section shows that temporary shocks to experience productivity growth $\widehat{g}_t$ are empirically very hard to distinguish from demand shocks such as monetary policy shocks or discount factor shocks. In particular, we show that

1. Typical sign-based identification approaches of demand shocks may actually identify shocks to experience productivity growth.

2. High-frequency-based identification of monetary policy shocks may also identify shocks to experience productivity growth.

These findings provide an alternative explanation for the somewhat puzzling empirical observation that measures of aggregate productivity increase following identified monetary policy loosenings. The literature attributes these productivity responses to the presence of steady-state misallocations that are temporarily reduced (aggravated) after positive (negative) demand shocks; see Meier and Reinelt (2024) and Baqaee et al. (2024). Our findings suggest that they could alternatively emerge because shocks to experience productivity growth are mistakenly interpreted as monetary policy shocks.

6.1 Pitfalls of Sign-Based Demand Shock Identification

To illustrate how sign-based identification approaches of demand shocks may actually also identify supply shocks, suppose the economy is at its deterministic steady state in $t = -1$. In $t = 0$, a purely temporary, positive shock to experience productivity growth $\widehat{g}_0 > 0$ hits the economy, as illustrated in the top panel of figure 2. The shock leads to a persistent fall in the natural real rate:[27]

$$r_t^n = -\rho^{t+1}(1-\rho) \cdot \widehat{g}_0, \tag{37}$$

Under optimal monetary policy, the nominal rate follows its natural level and simply remains at its steady-state value:

$$\hat{\imath}_t = \hat{\imath}_t^n = 0,$$

where the second equality follows from equation (36). Letting $\overline{\Gamma}_t^e$ denote the deterministic output level in the absence of the shock and $\widetilde{y}_t = \ln Y_t - \ln \overline{\Gamma}_t^e$ the log deviation from this deterministic level, we get[28]

$$\widetilde{y}_t = \rho^{t+1}\widehat{g}_0 = -\frac{1}{1-\rho} r_t^n,$$

which shows that output persistently increases when the natural real rate declines. The inflation response is given by[29]

$$\pi_t = \rho^t(1-\rho) \cdot \widehat{g}_0 = -\frac{1}{\rho} r_t^n$$

[27]This follows from equations (28) and (29).

[28]We focus on deviations of output from trend rather than on the output gap, because the latter is not directly observed in empirical work. Appendix B.5 shows how the path of detrended output can be computed. For the case here, where the output gap is closed, output simply evolves according to $\widetilde{y}_t = \widehat{\gamma}_t^e + \widetilde{y}_{t-1}$, where $\widehat{\gamma}_t^e$ is determined in equation (32).

[29]This follows from equation (28).

and also displays a persistent increase when the natural real rate declines.

Detrended aggregate productivity also increases in line with detrended output, but ultimately all variables return to their pre-shock value in detrended terms. Since the supply shock produces no long-run response for any of these variables, it satisfies the sign-based restrictions typically used in empirical demand shock identification, which require output and inflation to move in the same direction without displaying permanent effects; see Giannone and Primiceri (2024) and Bergholt et al. (2025) for recent examples.

Positive demand shocks, such as a monetary policy surprise loosening, also produce a persistent increase in output and inflation within our setup. Specifically, consider a setting in which monetary policy announces in $t = 0$ to implement a path for nominal rates that generates the same path (37) for the real rate as the supply shock. We then have the following result:

Lemma 2 *Suppose the economy is in steady state in $t = -1$ and monetary policy unexpectedly announces in $t = 0$ to implement a path for nominal rates that gives rise to the path of real interest rates (37). We then have*

$$\begin{pmatrix} \pi_t \\ \widetilde{y}_t \end{pmatrix} = \begin{pmatrix} -\frac{\kappa}{(1-\rho)(1-\beta\rho)} \\ -\frac{1}{(1-\rho)} \end{pmatrix} r_t^n. \tag{38}$$

The proof of lemma 2 can be found in appendix D. In line with conventional wisdom, temporarily lower real rates persistently increase output and inflation, without generating long-run effects. The difficulty for sign-based demand shock identification is that temporary experience growth shocks do the same, as illustrated above.

One difference between the effects of a temporary experience growth shock and the effects of a monetary policy surprise in lemma 2 is that the nominal rate falls in the latter case but stays constant in the former (under optimal policy). The following lemma shows that this difference is not key for our findings:

Lemma 3 *Suppose all agents learn in $t = 0$ about a sequence of preference shocks $\{\widehat{\xi}_t\}_{t=0}^{\infty}$*

that induces the natural rate path

$$r_t^n = \rho^{t+1}(1-\rho)\widehat{g}_0 - \frac{\kappa}{1-\beta\rho}\rho^{t+2}\widehat{g}_0, \tag{39}$$

but that monetary policy keeps nominal rates constant at the steady state value. The response of the economy is then again given by equation (38).

The proof of lemma 3 can be found in appendix E. Lemma 3 shows that preference shocks, which are prototypical demand shocks, persistently move output and inflation in the same direction, while nominal rates remain constant. The same is true for the experience growth shock. This illustrates that the risk of misidentifying supply shocks as demand shocks is high, when relying on commonly used sign-based identification restrictions.

6.2 Pitfalls of High-Frequency Monetary Policy Shock Identification

This section shows that high-frequency approaches to identify monetary policy shocks, as pioneered by Kuttner (2001), may also identify supply shocks as monetary policy shocks. We show that this is true even if one seeks to controls for so-called central bank information effects along the lines of Jarocinski and Karadi (2020).

To illustrate this point, consider an economy that is at its deterministic steady state in $t = -1$ and in which monetary policy is optimal. The latter implies that there are no monetary policy shocks: monetary policy responds only to fundamental shocks, but is not subject to random surprises.

In $t = 0$, the economy is hit by a positive shock to experience productivity growth which is partly reversed in future periods.[30] At the beginning of period $t = 0$, the central bank observes this shock, but the private sector initially does not. At the press conference, which takes place at the beginning of period $t = 0$ and shortly after the central bank learns about the shock, the central bank announces the shock to private agents and

[30] Specifically, we have $\widehat{g}_0 > 0$ and $\widehat{g}_t < 0$ for $t > 0$ with $-\sum_{t>0}\widehat{g}_t < \widehat{g}_0$.

also announces its optimal policy response.

Figure 6 illustrates the response of output, inflation, and nominal interest rates to such a supply shock.[31] It shows that nominal rates unexpectedly decrease, which follows directly from equation (36); output unexpectedly and persistently increases, due to increased experience productivity; inflation unexpectedly increases, as inflation follows its optimal target, see equation (28). Clearly, the impulse response in figure 6 mimics that of a random monetary policy surprise loosening, as considered in the previous section. However, the policy surprise is instead the result of (i) new information revealed by the central bank, and (ii) the systematic response of monetary policy and the private sector to this new information.

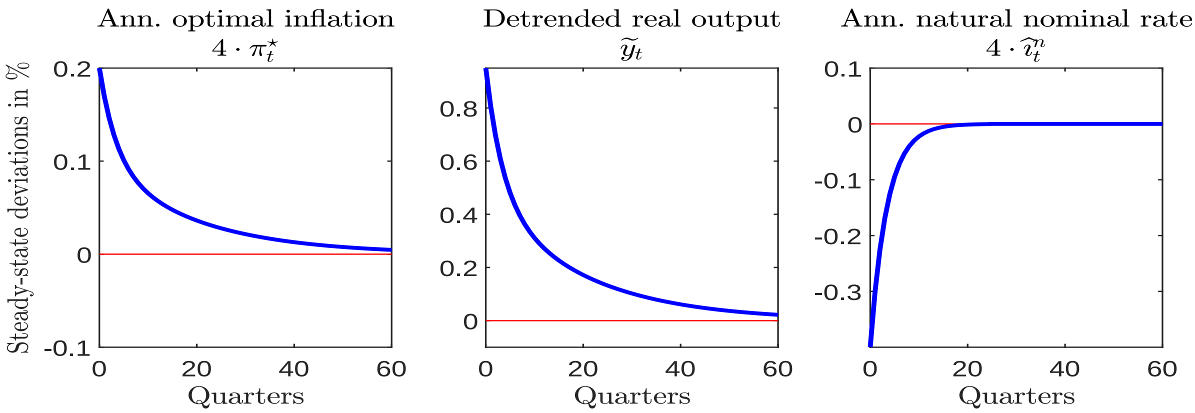

Figure 6: Impulse responses of a temporary and partly reversed shock to $\widehat{g}_t$.

It is well-known to empirical researchers that central bank press conferences reveal information about the state of the economy. Therefore, it has become standard in the high-frequency identification of monetary policy shocks to filter out so-called central bank 'information effects'. Following Jarocinski and Karadi (2020), much of the empirical literature thus requires that a surprise loosening of monetary policy is associated with a positive stock market response.

However, this approach is ineffective here because the surprise loosening following the experience growth shock is associated with a positive stock market response, as we show below. Thus, standard high-frequency identification approaches would erroneously

[31] The figure assumes $\rho = 0.95$ and considers the shock $\widehat{g}_0 = 0.01$, $\widehat{g}_t = -0.1 \cdot (0.75)^t \cdot 0.01$ for $1 \leq t \leq 24$ and $\widehat{g}_t = 0$ for $t > 24$.

classify the response to the supply shock in figure 6 as a random monetary policy surprise.

To understand why the stock market appreciates, note that real output temporarily increases in figure 6 due to the temporary increase in aggregate productivity. Real firm profits increase proportionately to real output;[32] in addition, the real interest rate falls, so that the present value of real discounted profits increases. The stock market thus appreciates in value.

It is worth repeating that the considered supply shock also increases aggregate productivity. This may offer a complementary explanation for the empirical finding that measured productivity rises following identified monetary policy surprise loosenings, even though productivity in the model does not respond to first order following a true monetary policy shock, e.g., the one considered in lemma 2.

6.3 Potential Remedies to Misidentification

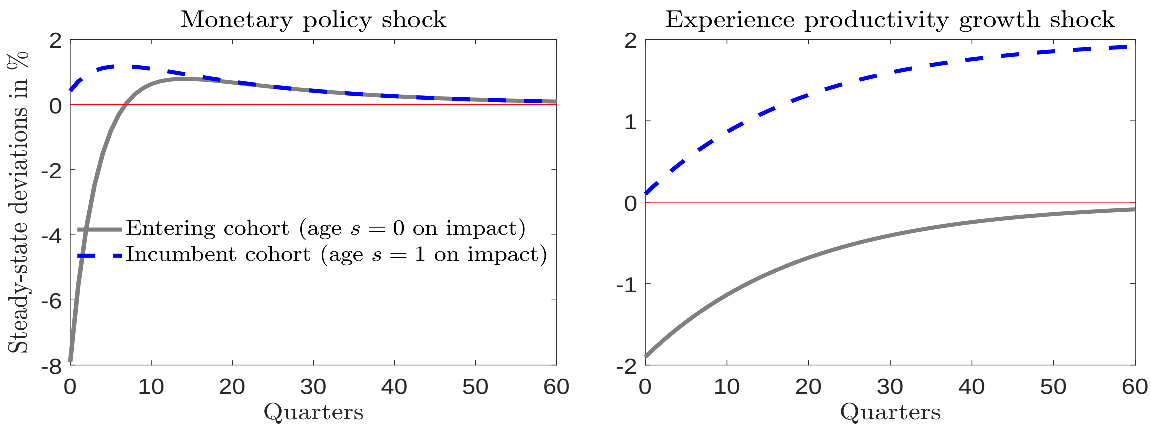

Figure 7: Cohort-level expenditure shares: Monetary policy loosening shock (left) and positive experience productivity growth shock (right)

The previous sections illustrated the potential to misidentify supply shocks as demand shocks when relying on aggregate variables. This section shows that even micro-level information may fail to overcome the misidentification problem.

Figure 7 depicts impulse responses of the expenditure shares for different firm cohorts. The left panel shows the impulse response following the monetary policy shock considered

[32]Appendix B.6 shows that real profits move proportionately to real output when the output gap is closed and there are no mark-up shocks, as is the case considered here.

in lemma 2, while the right panel shows the impulse response following the temporary experience productivity growth shock $\widehat{g}_0 > 0$ considered at the beginning of section 6.1.[33] The figure contrasts the expenditure share response for firms that newly enter the economy in the period of the shock to the response of firms that entered in the period prior to the shock.[34] The latter firms have sticky prices while the former set prices flexibly. The response of older firms closely tracks that of firms that entered in the period prior to the shock and thus is not depicted.

For both shock scenarios, new firms lose market share and older firms gain market share on impact, so that the sign of the impact response cannot be used to distinguish between a true monetary policy loosening and the optimal response to an experience productivity growth shock. The only difference between the two shocks is that the divergence in expenditure shares is short-lived following the monetary policy shock, whereas experience productivity shocks create a permanent divergence in expenditure shares. However, the latter is no longer true when experience growth shocks partially reverse over time, a situation considered in section 6.2. It is thus difficult to empirically distinguish experience growth shocks from true monetary policy shocks, even when considering firm-level responses to these shocks.

7 Conclusions and Outlook

Introducing a firm (or product) life cycle into New Keynesian models generates several interesting policy implications: the optimal inflation target is nonzero in steady state and varies over time following supply disturbances that alter the relative productivity between new and incumbent firms; optimal monetary policy "looks through" these supply disturbances, unlike in a model without turnover; and the identification of demand shocks is more challenging than in models without turnover because some supply disturbances move output and inflation in the same direction under optimal policy.

These implications raise new challenges for monetary policy. Implementing optimal

[33] Appendix F describes the computation of cohort-level impulse responses underlying figure 7. The figure assumes parameters $\theta = 3, \kappa = 0.1, \alpha = 0.75, \rho = 0.95$ and $\beta = 0.99$ and shock $\widehat{g}_0 = 0.01$.

[34] All expenditure shares are shown in percent deviation from their steady-state shares.

policy requires that policymakers distinguish between traditional mark-up disturbances in the Phillips curve, which generate a trade-off between output gap and inflation gap stabilization, and a new kind of Phillips curve disturbance, which captures movements in the welfare-optimal inflation target and does not give rise to such a trade-off.

The model with supply-side turnover also implies that the optimal inflation target is affected by structural change in the economy. Declining business dynamism in the form of lower firm turnover, for instance, implies that movements in the inflation target become more persistent over time. The emergence of innovative productivity-enhancing technologies, such as artificial intelligence, affects the inflation target in ways that depend on whether these technologies enter the economy predominantly through new or existing firms. The potentially changing nature of the optimal inflation target implies that monetary policy institutions are well advised to conduct regular framework reviews that reassess whether their targets are still appropriate.

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Appendix to
"Monetary Policy and Supply-Side Turnover"

by Klaus Adam and Henning Weber

2025 Econometric Society World Congress Volume

A Economic Model Setup

We consider the model in Adam and Weber (2019), augment it with a time-varying sales subsidy, and derive the linear-quadratic approximation to the optimal monetary policy problem. Importantly, we consider in this and the subsequent appendices a setup that generalizes the one in the main text of this chapter, as we allow also for the indexation of prices to past inflation during periods in which prices are not set optimally.

To obtain a formulation of the problem that nests the one in the canonical model without firm turnover, price indexation takes the form

$$P_{jt+1} = P_{jt} \left(\frac{\Pi_t}{\Pi^\star} \right)^{\varphi},$$

for all firms that do not reoptimize prices, where $\varphi \in [0, 1]$ is the indexation parameter. Price indexation only reacts to deviations of actual inflation from its optimal steady-state value $\Pi^\star$. For the special case where optimal steady-state inflation $\Pi^\star = 1$ and $\varphi = 1$, the scheme reduces to the one considered in Christiano, Eichenbaum and Evans (2005). For $\varphi = 0$, one recovers a setup without price indexation, as considered in the main text. We then prove the following generalized result:

Proposition 4 *Suppose Assumption 1 holds. A linear-quadratic approximation to the* *optimal monetary policy problem is then given by*

$$\max_{\{x_t,\pi_t,\hat{\imath}_t\}_{t=0}^{\infty}} -E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\pi_t - \pi_t^\star - \varphi\left(\pi_{t-1} - \pi_{t-1}^\star\right)\right)^2 + \lambda x_t^2 \right] \tag{40}$$

$$s.t.:$$

$$\pi_t - \pi_t^\star - \varphi\left(\pi_{t-1} - \pi_{t-1}^\star\right) = \beta E_t\left[\pi_{t+1} - \pi_{t+1}^\star - \varphi\left(\pi_t - \pi_t^\star\right)\right] + \kappa x_t + u_t$$

$$x_t = E_t x_{t+1} - \left(\hat{\imath}_t - E_t \pi_{t+1}\right) + r_t^n,$$

with π_{-1} *given,* $\lambda \equiv \kappa/\theta$*, and*

$$\pi_t^\star - \varphi\pi_{t-1}^\star = \rho\left(\pi_{t-1}^\star - \varphi\pi_{t-2}^\star\right) + (1-\rho)\cdot\left(\widehat{g}_t - \widehat{q}_t\right),$$

with $\pi_{-1}^\star$ *and* $\pi_{-2}^\star$ *given, and*

$$\rho \equiv (1-\delta)\left(\Pi^\star\right)^{(\theta-1)} < 1.$$

The natural rate evolves according to

$$r_t^n = E_t\left[\widehat{a}_{t+1} + \widehat{g}_{t+1} - \widehat{\xi}_{t+1} - \left(\pi_{t+1}^\star - \varphi\pi_t^\star\right)\right]. \tag{41}$$

The next sections derive key equations of the generalized model, while appendices B and C prove the previous proposition.

A.1 Price Setting with Time-Varying Sales Subsidy

The firm that can optimize its price in period t solves the following problem:

$$\max_{P_{jt}} E_t \sum_{i=0}^{\infty} (\alpha(1-\delta))^i \Omega_{t,t+i} \left((1+\tau_{t+i}) \frac{P_{jt+i}}{P_{t+i}} Y_{jt+i} - \frac{W_{t+i}}{P_{t+i}} \frac{Y_{jt+i}}{A_{t+i} Q_{t-s_{jt}} G_{jt+i}} \right) \tag{42}$$

$$s.t. \quad Y_{jt+i} = \left(P_{jt+i}/P_{t+i}\right)^{-\theta} Y_{t+i},$$

$$P_{jt+i+1} = \Xi_{t+i,t+i+1} P_{jt+i},$$

where we substituted firm technology (3) and firm productivity $Z_{jt} = G_{jt} Q_{t-s_{jt}}$ into the firm's period profits. In equation (42), α denotes the Calvo probability that the firm has to keep its previous price ($0 \leq \alpha < 1$), τ_{t+i} denotes the sales subsidy and $\Omega_{t,t+i} = \beta \frac{\xi_{t+i}}{\xi_t} \frac{U_{Ct+i}}{U_{Ct}}$ denotes the household's discount factor between time t and $t+i$. The first constraint in the price-setting problem captures the firm's demand function, see equation (8), and the second constraint captures that the firm's price is indexed between time t and $t+1$ according to $\Xi_{t,t+1} = (\Pi_t/\Pi^\star)^{\varphi}$ in periods in which prices are not reset optimally. The price-setting problem in equation (42) implies that the optimal product price is given by

$$P^\star_{jt} = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}Y_{t+i}\left(\Xi_{t,t+i}/P_{t+i}\right)^{-\theta}\frac{W_{t+i}/P_{t+i}}{A_{t+i}Q_{t-s_{jt}}G_{jt+i}}}{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}Y_{t+i}\left(\frac{1+\tau_{t+i}}{1+\tau}\right)\left(\Xi_{t,t+i}/P_{t+i}\right)^{1-\theta}},$$

where the cumulative effect of price indexation is given by the product $\Xi_{t,t+i} = \prod_{k=1}^{i}\Xi_{t+k-1,t+k}$. Rewriting the previous equation using analogous steps as in Appendix A.2 (Price-Setting Problem of Firms) in Adam and Weber (2019) yields

$$\frac{P^\star_{jt}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right) = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t},$$

where numerator N_t and denominator D_t are given by

$$N_t = E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{-\theta}\frac{W_{t+i}}{P_{t+i}A_{t+i}Q_{t+i}}\left(\frac{q_{t+i}\times\cdots\times q_{t+1}}{g_{t+i}\times\cdots\times g_{t+1}}\right)$$
$$D_t = E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{1-\theta}\left(\frac{1+\tau_{t+i}}{1+\tau}\right).$$

Both variables can be expressed recursively according to

$$N_t = \frac{W_t}{P_tA_tQ_t} + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta}\left(\frac{q_{t+1}}{g_{t+1}}\right)N_{t+1}\right] \tag{43}$$
$$D_t = \frac{1+\tau_t}{1+\tau} + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta-1}D_{t+1}\right]. \tag{44}$$

A.2 Equilibrium Conditions

In the model of Adam and Weber (2019), we assume that utility is given by equation (1) following Christiano, Trabandt and Walentin (2010), there is no capital accumulation ($\phi = 1$) and fixed costs of production are equal to zero ($f = 0$). Under these assumptions, and when accounting for the time-varying sales subsidy, the equilibrium conditions in Appendix A.7 (Transformed Sticky Price Economy) in Adam and Weber (2019) are as follows:

$$1 = \left[\alpha\delta + (1-\alpha)(\Delta_t^e)^{1-\theta}\right](p_t^\star)^{1-\theta} + \alpha(1-\delta)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta-1} \tag{45}$$

$$\Delta_t = \left[\alpha\delta + (1-\alpha)(\Delta_t^e)^{1-\theta}\right](p_t^\star)^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\Delta_{t-1} \tag{46}$$

$$p_t^\star = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t} \tag{47}$$

$$N_t = \frac{w_t}{\Delta_t^e} + \alpha(1-\delta)\beta E_t\left[\left(\frac{\xi_{t+1}}{\xi_t}\right)\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta}\left(\frac{q_{t+1}}{g_{t+1}}\right)N_{t+1}\right] \tag{48}$$

$$D_t = \frac{1+\tau_t}{1+\tau} + \alpha(1-\delta)\beta E_t\left[\left(\frac{\xi_{t+1}}{\xi_t}\right)\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta-1}D_{t+1}\right] \tag{49}$$

$$y_t = \frac{\Delta_t^e}{\Delta_t}L_t \tag{50}$$

$$\gamma_t^e = a_t q_t \Delta_{t-1}^e/\Delta_t^e \tag{51}$$

$$(\Delta_t^e)^{1-\theta} = \delta + (1-\delta)\left(\Delta_{t-1}^e q_t/g_t\right)^{1-\theta} \tag{52}$$

$$w_t = y_t L_t^{\psi} \tag{53}$$

$$1 = \beta E_t\left[\left(\frac{\xi_{t+1}}{\xi_t}\right)\left(\frac{\gamma_{t+1}^e y_{t+1}}{y_t}\right)^{-1}\left(\frac{1+i_t}{\Pi_{t+1}}\right)\right] \tag{54}$$

$$\Xi_{t-1,t} = (\Pi_{t-1}/\Pi^\star)^{\varphi} \tag{55}$$

We denote the variables $p_t^\star = P_{t,t}^\star/P_t$, $\Gamma_t^e = A_t Q_t/\Delta_t^e$, $w_t = W_t/(P_t\Gamma_t^e)$ and $y_t = Y_t/\Gamma_t^e$. The variable $\Xi_{t-1,t}$ captures either price indexation to lagged inflation, with $\varphi > 0$, or no price indexation, with $\varphi = 0$.

A.3 Efficient Level of Output, Hours and Real Rate

Equations (45) to (55) nest the efficient equilibrium in the special case with flexible prices, $\alpha = 0$, and a sales subsidy that fulfills equation (14). We denote variables in the efficient equilibrium with superscript e. Under these assumptions, equation (45) yields

$$p_t^e = 1/\Delta_t^e, \tag{56}$$

and equations (47), (48) and (49) imply $N_t^e = w_t^e/\Delta_t^e$, $D_t^e = 1$ and hence

$$w_t^e = 1, \tag{57}$$

also using that $p_t^\star = p_t^e$. Equations (50) and (53) then imply that the efficient level of detrended output and the efficient level of hours worked are given by

$$y_t^e = L_t^e = 1. \tag{58}$$

The efficient (gross) real rate follows from equation (54) and is given by

$$\frac{1}{R_t^e} = \beta E_t\left(\frac{\xi_{t+1}}{\xi_t}\frac{1}{\gamma_{t+1}^e}\right). \tag{59}$$

A.4 Sticky Price Economy Expressed in Gaps

Substituting equation (56) into equation (46) and rearranging yields

$$\begin{aligned}\frac{\Delta_t}{\Delta_t^e} = {} & \left[\alpha\delta(\Delta_t^e)^{\theta-1} + (1-\alpha)\right]\left(\frac{p_t^\star}{p_t^e}\right)^{-\theta} \\ & + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\left(\frac{\Delta_{t-1}}{\Delta_{t-1}^e}\right)\frac{\Delta_{t-1}^e}{\Delta_t^e}\ .\end{aligned} \tag{60}$$

Proposition 1 in Adam and Weber (2019) shows that

$$\frac{\Pi_t^\star}{\Xi_{t-1,t}^\star} = \left(\frac{1-\delta\left(\Delta_t^e\right)^{\theta-1}}{1-\delta}\right)^{\frac{1}{\theta-1}}\ . \tag{61}$$

We use this equation to rearrange the term in square brackets in equation (60), which yields

$$\frac{\Delta_t}{\Delta_t^e} = \left[1 - \alpha(1-\delta)\left(\frac{\Pi_t^\star}{\Xi_{t-1,t}^\star}\right)^{\theta-1}\right]\left(\frac{p_t^\star}{p_t^e}\right)^{-\theta}$$
$$+ \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\left(\frac{\Delta_{t-1}}{\Delta_{t-1}^e}\right)\frac{\Delta_{t-1}^e}{\Delta_t^e}. \tag{62}$$

From equations (52) and (61), we also obtain that

$$\frac{\Delta_{t-1}^e}{\Delta_t^e} = \frac{g_t}{q_t}\left(\frac{1}{\Pi_t^\star/\Xi_{t-1,t}^\star}\right), \tag{63}$$

and we use this equation to rearrange equation (62), which yields

$$\frac{\Delta_t}{\Delta_t^e} = \left[1 - \alpha(1-\delta)\left(\frac{\Pi_t^\star}{\Xi_{t-1,t}^\star}\right)^{\theta-1}\right]\left(\frac{p_t^\star}{p_t^e}\right)^{-\theta}$$
$$+ \alpha(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}\left(\frac{\Pi_t/\Xi_{t-1,t}}{\Pi_t^\star/\Xi_{t-1,t}^\star}\right)^{\theta}\left(\frac{\Delta_{t-1}}{\Delta_{t-1}^e}\right). \tag{64}$$

Furthermore, we use equation (61) to rearrange the term in square brackets in equation (45), and use equation (56) to substitute p_t^e into equation (45). This yields

$$\left(\frac{p_t^\star}{p_t^e}\right)^{1-\theta} = \frac{1 - \alpha(1-\delta)(\Pi_t/\Xi_{t-1,t})^{\theta-1}}{1 - \alpha(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}}. \tag{65}$$

We also divide equation (47) by p_t^e, impose $1 = \frac{\theta}{\theta-1}\frac{1}{1+\tau}$, multiply equation (48) by Δ_t^e, and denote $\widetilde{N}_t = N_t\Delta_t^e$ to obtain

$$\frac{p_t^\star}{p_t^e} = \frac{\widetilde{N}_t}{D_t}, \tag{66}$$

$$\widetilde{N}_t = w_t + \alpha(1-\delta)\beta E_t\left[\left(\frac{\Pi_{t+1}^\star}{\Xi_{t,t+1}^\star}\right)^{\theta-1}\left(\frac{\Pi_{t+1}/\Xi_{t,t+1}}{\Pi_{t+1}^\star/\Xi_{t,t+1}^\star}\right)^{\theta}\left(\frac{\xi_{t+1}}{\xi_t}\right)\widetilde{N}_{t+1}\right], \tag{67}$$

$$D_t = \frac{1+\tau_t}{1+\tau} + \alpha(1-\delta)\beta E_t\left[\left(\frac{\Pi_{t+1}^\star}{\Xi_{t,t+1}^\star}\right)^{\theta-1}\left(\frac{\Pi_{t+1}/\Xi_{t,t+1}}{\Pi_{t+1}^\star/\Xi_{t,t+1}^\star}\right)^{\theta-1}\left(\frac{\xi_{t+1}}{\xi_t}\right)D_{t+1}\right], \tag{68}$$

where we also used equation (63) to rewrite $\widetilde{N}_t$ and rewrite the equation for D_t correspondingly.

B Approximate Equilibrium Conditions

We linearize the sticky-price equilibrium conditions at the efficient steady state, in which $\Pi = \Pi^\star = g/q$, $\Xi = \Xi^\star = 1$ and the sales subsidy fulfills equation (14).

B.1 Optimal Inflation Target

Rearranging equation (61) yields $(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1} = 1 - \delta\,(\Delta_t^e)^{\theta-1}$, and linearizing this equation in the variables $\Pi_t^\star/\Xi_{t-1,t}^\star$ and Δ_t^e yields

$$(1-\delta)(\Pi^\star/\Xi^\star)^{\theta-1}\widetilde{\pi}_t^\star = -\delta\,(\Delta^e)^{\theta-1}\,\widehat{\Delta}_t^e,$$

after defining

$$\widetilde{\pi}_t^\star = \ln(\Pi_t^\star/\Xi_{t-1,t}^\star) - \ln(\Pi^\star/\Xi^\star).$$

Using the steady-state equation $(1-(1-\delta)(\Pi^\star/\Xi^\star)^{\theta-1}) = \delta\,(\Delta^e)^{\theta-1}$ thus yields

$$\rho\widetilde{\pi}_t^\star = -(1-\rho)\widehat{\Delta}_t^e, \tag{69}$$

after using the definition of ρ in Proposition 1 given by

$$\rho \equiv (1-\delta)(\Pi^\star)^{\theta-1},$$

which also applies to the case with price indexation given that $\Xi^\star = 1$. Linearizing equation (63) yields

$$\widehat{\Delta}_{t-1}^e - \widehat{\Delta}_t^e = \widehat{g}_t - \widehat{q}_t - \widetilde{\pi}_t^\star, \tag{70}$$

and multiplying by $(1-\rho)$ and rearranging yields

$$-(1-\rho)\widehat{\Delta}_t^e = -(1-\rho)\widehat{\Delta}_{t-1}^e + (1-\rho)(\widehat{g}_t - \widehat{q}_t - \widetilde{\pi}_t^\star).$$

Substituting for $-(1-\rho)\widehat{\Delta}_t^e$ and $-(1-\rho)\widehat{\Delta}_{t-1}^e$ using equation (69) and simplifying terms yields

$$\widetilde{\pi}_t^\star = \rho\widetilde{\pi}_{t-1}^\star + (1-\rho)(\widehat{g}_t - \widehat{q}_t). \tag{71}$$

In the case without price indexation, we obtain $\widetilde{\pi}_t^\star = \pi_t^\star$ and thus

$$\pi_t^\star = \rho \pi_{t-1}^\star + (1-\rho)(\widehat{g}_t - \widehat{q}_t). \tag{72}$$

B.2 New Keynesian Phillips Curve

Taking logs of equation (66) and using that $\widetilde{N}^e = D^e$ in steady state yields

$$\widehat{p}_t^\star = \ln(\widetilde{N}_t/\widetilde{N}^e) - \ln(D_t/D^e), \tag{73}$$

denoting $\widehat{p}_t^\star = \ln(p_t^\star/p_t^e) - \ln(p^\star/p^e)$, which is equal to

$$\widehat{p}_t^\star = \ln p_t^\star - \ln p_t^e \tag{74}$$

since $p^\star/p^e = 1$. Linearizing equations (67) and (68) and subtracting the linearized equations from each other yields

$$\begin{aligned}\ln(\widetilde{N}_t/\widetilde{N}^e) - \ln(D_t/D^e) = (1-\alpha\rho\beta)\left(\ln w_t - \ln\frac{1+\tau_t}{1+\tau}\right)\\ + \alpha\rho\beta E_t[\ln(\widetilde{N}_{t+1}/\widetilde{N}^e) - \ln(D_{t+1}/D^e) + \widetilde{\pi}_{t+1} - \widetilde{\pi}_{t+1}^\star] ,\end{aligned}$$

where we define

$$\widetilde{\pi}_t \equiv \ln(\Pi_t/\Xi_{t-1,t}) - \ln(\Pi^\star/\Xi^\star) . \tag{75}$$

Using equation (73) to substitute $\widehat{p}_t^\star$ for $\ln(\widetilde{N}_t/\widetilde{N}^e) - \ln(D_t/D^e)$ in the previous equation yields

$$\widehat{p}_t^\star = (1-\alpha\rho\beta)\left(\ln w_t - \ln\frac{1+\tau_t}{1+\tau}\right) + \alpha\rho\beta E_t[\widehat{p}_{t+1}^\star + \widetilde{\pi}_{t+1} - \widetilde{\pi}_{t+1}^\star] . \tag{76}$$

Linearizing equation (65) yields

$$\widehat{p}_t^\star = \frac{\alpha\rho}{1-\alpha\rho}(\widetilde{\pi}_t - \widetilde{\pi}_t^\star) . \tag{77}$$

Substituting this equation into equation (76) and simplifying terms yields

$$\widetilde{\pi}_t - \widetilde{\pi}_t^\star = \frac{(1-\alpha\rho)(1-\alpha\rho\beta)}{\alpha\rho}\left(\ln w_t - \ln\frac{1+\tau_t}{1+\tau}\right) + \beta E_t[\widetilde{\pi}_{t+1} - \widetilde{\pi}_{t+1}^\star] . \tag{78}$$

Taking logs of equation (50) and using that the log of the productivity-adjusted dispersion gap is a second-order term provided its initial value is of second order, see equation (119), yields

$$\ln y_t = \ln L_t. \tag{79}$$

Taking logs of equation (53) and substituting equation (79) yields

$$\ln w_t = (1+\psi)\ln y_t = (1+\psi)x_t, \tag{80}$$

where the second equality follows because

$$x_t = \ln y_t = \ln Y_t - \ln Y_t^e,$$

since $y_t = Y_t/\Gamma_t^e = Y_t/Y_t^e$ from equation (58). Substituting equation (80) into equation (78) yields the New Keynesian Phillips curve

$$\widetilde{\pi}_t - \widetilde{\pi}_t^\star = \kappa x_t + \beta E_t[\widetilde{\pi}_{t+1} - \widetilde{\pi}_{t+1}^\star] + u_t, \tag{81}$$

where the markup disturbance and the slope of this curve are defined in equations (22) and (23), respectively.

B.3 Price Indexation Equation

Multiplying the inverse of equation (55) by gross inflation, taking logs, subtracting $\ln(\Pi^\star/\Xi^\star)$ and using that $\Xi^\star = 1$ yields

$$\ln(\Pi_t/\Xi_{t-1,t}) - \ln(\Pi^\star/\Xi^\star) = \ln(\Pi_t/\Pi^\star) - \varphi\ln(\Pi_{t-1}/\Pi^\star). \tag{82}$$

Exploiting definitions (75) and (16) further yields

$$\widetilde{\pi}_t = \pi_t - \varphi\pi_{t-1}. \tag{83}$$

Analogously, we obtain

$$\widetilde{\pi}_t^\star = \pi_t^\star - \varphi\pi_{t-1}^\star. \tag{84}$$

B.4 Euler Equation and Natural Real Rate

Linearizing equation (54) yields

$$0 = E_t[\widehat{\xi}_{t+1} + \widehat{y}_t - \widehat{y}_{t+1} - \widehat{\gamma}^e_{t+1} + \widehat{\imath}_t - \pi_{t+1}],$$

where $\widehat{\xi}_{t+1}$ denotes the growth rate of the time preference shock in equation (1), π_{t+1} is defined in equation (16) and $\hat{\imath}_t$ is defined in equation (21) using that $1 + i = ag/\beta$. Also using that $\widehat{y}_t = x_t$ since $\widehat{y}_t = \ln y_t = \ln Y_t - \ln \Gamma^e_t = \ln Y_t - \ln Y^e_t$ which is the output gap defined in equation (19), yields

$$x_t = E_t[x_{t+1} - (\hat{\imath}_t - \pi_{t+1}) + \widehat{\gamma}^e_{t+1} - \widehat{\xi}_{t+1}]. \tag{85}$$

Linearizing equation (59) yields

$$r^n_t = E_t[\widehat{\gamma}^e_{t+1} - \widehat{\xi}_{t+1}], \tag{86}$$

where r^n_t is defined in equation (20). Substituting the previous equation into equation (85) yields the Euler equation

$$x_t = E_t x_{t+1} - (\hat{\imath}_t - E_t \pi_{t+1} - r^n_t). \tag{87}$$

B.5 Efficient Output Growth and Detrended Output

Linearizing equation (51) yields

$$\widehat{\gamma}^e_t = \widehat{a}_t + \widehat{q}_t + \widehat{\Delta}^e_{t-1} - \widehat{\Delta}^e_t,$$

and substituting for $\widehat{\Delta}^e_{t-1} - \widehat{\Delta}^e_t$ using equation (70) and simplifying yields

$$\widehat{\gamma}^e_t = \widehat{a}_t + \widehat{g}_t - \widetilde{\pi}^\star_t. \tag{88}$$

To obtain a representation of $\widehat{\gamma}^e_t$ in terms of supply disturbances only, we rearrange the previous equation to obtain $\widetilde{\pi}^\star_t = -(\widehat{\gamma}^e_t - \widehat{a}_t - \widehat{g}_t)$, and substitute this expression into equation (71). After rearranging terms, this yields

$$\widehat{\gamma}^e_t = \rho\widehat{\gamma}^e_{t-1} + (\widehat{a}_t - \rho\widehat{a}_{t-1}) + \rho(\widehat{g}_t - \widehat{g}_{t-1}) + (1 - \rho)\widehat{q}_t. \tag{89}$$

To relate efficient output growth to detrended output, note that efficient output growth at the balanced growth path is given by $\gamma^e = \overline{\Gamma}^e_t/\overline{\Gamma}^e_{t-1}$, where the aggregate growth trend at this path evolves according to

$$\overline{\Gamma}^e_t = \bar{A}_t\bar{Q}_t/\Delta^e,$$

with $\bar{A}_t = a \cdot \bar{A}_{t-1}$, $\bar{Q}_t = q \cdot \bar{Q}_{t-1}$ and where $1/\Delta^e$ denotes the endogenous component of aggregate productivity in the efficient steady state. This implies

$$\widehat{\gamma}^e_t = \ln(\gamma^e_t/\gamma^e) = \ln(\Gamma^e_t/\overline{\Gamma}^e_t) - \ln(\Gamma^e_{t-1}/\overline{\Gamma}^e_{t-1}).$$

Augmenting the two RHS variables by output and rearranging yields

$$\ln(Y_t/\overline{\Gamma}^e_t) = \widehat{\gamma}^e_t + \ln(Y_{t-1}/\overline{\Gamma}^e_{t-1}) + \ln(Y_t/\Gamma^e_t) - \ln(Y_{t-1}/\Gamma^e_{t-1}).$$

Using the definition $\widetilde{y}_t = \ln(Y_t/\overline{\Gamma}^e_t)$ and that $Y^e_t = \Gamma^e_t$, which again follows from equation (58), we obtain for the previous equation that

$$\widetilde{y}_t = \widehat{\gamma}^e_t + \widetilde{y}_{t-1} + \ln(Y_t/Y^e_t) - \ln(Y_{t-1}/Y^e_{t-1}).$$

Using the definition of the output gap in equation (19) shows that log-linear detrended output evolves according to

$$\widetilde{y}_t = \widehat{\gamma}^e_t + \widetilde{y}_{t-1} + x_t - x_{t-1}. \tag{90}$$

We now consider the effect of a one-time experience growth shock, $\widehat{g}_0 > 0$, on $\widetilde{y}_t$ in a situation in which the output gap is closed, $x_t = 0$. With $x_t = 0$, the previous equation reduces to $\widetilde{y}_t = \widehat{\gamma}^e_t + \widetilde{y}_{t-1}$, and cumulating it over time yields

$$\widetilde{y}_t = \sum_{k=0}^{t} \widehat{\gamma}^e_k + \widetilde{y}_{-1}. \tag{91}$$

With only experience growth shocks, equation (89) reduces to

$$\widehat{\gamma}^e_t = \rho\widehat{\gamma}^e_{t-1} + \rho(\widehat{g}_t - \widehat{g}_{t-1}).$$

With $\widehat{g}_0 > 0$ and initial values $\widetilde{y}_{-1} = \widehat{\gamma}^e_{-1} = \widehat{g}_{-1} = 0$, this equation implies

$$\widehat{\gamma}^e_t = \begin{cases} \rho\widehat{g}_0, & t = 0, \\ -(1-\rho)\rho^t\widehat{g}_0, & t \geq 1. \end{cases}$$

Substituting this path for $\widehat{\gamma}^e_k$ into equation (91) delivers the effect on detrended output according to

$$\begin{aligned} \widetilde{y}_t &= \widehat{\gamma}^e_0 + \sum_{k=1}^{t} \widehat{\gamma}^e_k = \left(\rho - (1-\rho)\sum_{k=1}^{t}\rho^k\right)\widehat{g}_0 \\ &= \rho^{t+1}\widehat{g}_0, \end{aligned}$$

as stated in the main text.

B.6 Firm Profits

Nominal firm profits are given by

$$\Theta_t = (1+\tau_t)P_t Y_t - W_t L_t,$$

which implies that real firm profits Θ_t/P_t feature the same growth trend Γ^e_t as output Y_t and the real wage W_t/P_t. Defining detrended real profits as

$$\vartheta_t \equiv \frac{\Theta_t/P_t}{\Gamma^e_t},$$

the previous equation yields

$$\vartheta_t = (1+\tau_t)y_t - w_t L_t, \tag{92}$$

where as before, y_t and w_t denote detrended output and the detrended real wage, respectively. Detrended real profits in the efficient equilibrium, in which $\tau_t = \tau$, are given by

$$\vartheta^e_t = (1+\tau)y^e_t - w^e_t L^e_t.$$

Using equations (57) and (58) and the condition $1+\tau=\frac{\theta}{\theta-1}$, it follows that efficient detrended real profits are constant over time and equal to

$$\vartheta^e=\frac{1}{\theta-1}.$$

Linearizing equation (92) at the efficient steady state yields

$$\widehat{\vartheta}_t=(1+\tau)\frac{y^e}{\vartheta^e}\left(\widehat{y}_t-\frac{1+\psi}{\kappa}u_t\right)-\frac{w^eL^e}{\vartheta^e}(\widehat{w}_t+\widehat{L}_t), \tag{93}$$

after using the definition (22) and denoting a variable with a hat as the percentage deviation from its value in the efficient steady state. Using $1/\vartheta^e=\theta-1$, $1+\tau=\frac{\theta}{\theta-1}$ and $y^e=w^e=L^e=1$, equation (93) yields

$$\widehat{\vartheta}_t=\theta\left(\widehat{y}_t-\frac{1+\psi}{\kappa}u_t\right)-(\theta-1)(\widehat{w}_t+\widehat{L}_t). \tag{94}$$

Using $\widehat{y}_t=\widehat{L}_t$ from equation (79), $\widehat{w}_t=(1+\psi)\widehat{y}_t$ from equation (80) and $\widehat{y}_t=x_t$ implies that

$$\widehat{\vartheta}_t=[1-(\theta-1)(1+\psi)]x_t-\theta\left(\frac{1+\psi}{\kappa}\right)u_t, \tag{95}$$

with $\widehat{\vartheta}_t=\ln\vartheta_t-\ln\vartheta^e$. In situations without markup shocks $u_t=0$ and a closed output gap $x_t=0$, the previous equation implies $\widehat{\vartheta}_t=0$, hence $\vartheta_t=1/(\theta-1)$ and thus

$$\Theta_t/P_t=\frac{1}{\theta-1}\cdot\Gamma_t^e, \tag{96}$$

which shows that real profits after such shocks are proportional to the aggregate growth trend and, since $Y_t^e=\Gamma_t^e$, efficient output.

B.7 Approximate Equilibrium Conditions: Summary

The equilibrium in the approximate sticky-price economy is described by the equations (71), (81), (83), (84), (86), (87), (89), (90) and (95):

$$
\begin{aligned}
\widetilde{\pi}_t - \widetilde{\pi}^{\star}_t &= \kappa x_t + \beta E_t[\widetilde{\pi}_{t+1} - \widetilde{\pi}^{\star}_{t+1}] + u_t \\
x_t &= E_t x_{t+1} - (\widehat{\imath}_t - E_t \pi_{t+1} - r^n_t) \\
\widetilde{\pi}^{\star}_t &= \rho \widetilde{\pi}^{\star}_{t-1} + (1-\rho)(\widehat{g}_t - \widehat{q}_t) \\
r^n_t &= E_t[\widehat{\gamma}^e_{t+1} - \widehat{\xi}_{t+1}] \\
\widehat{\gamma}^e_t &= \rho \widehat{\gamma}^e_{t-1} + (\widehat{a}_t - \rho \widehat{a}_{t-1}) + \rho(\widehat{g}_t - \widehat{g}_{t-1}) + (1-\rho)\widehat{q}_t \\
\widetilde{y}_t &= \widehat{\gamma}^e_t + \widetilde{y}_{t-1} + x_t - x_{t-1} \\
\widetilde{\pi}_t &= \pi_t - \varphi \pi_{t-1} \\
\widetilde{\pi}^{\star}_t &= \pi^{\star}_t - \varphi \pi^{\star}_{t-1} \\
\widehat{\vartheta}_t &= [1 - (\theta - 1)(1+\psi)] x_t - \theta \left(\frac{1+\psi}{\kappa} \right) u_t,
\end{aligned}
$$

and a specification of monetary policy.

C Approximate Welfare-Based Loss Function

We denote expected discounted lifetime utility of the representative household in equation (1) according to

$$W = E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\ln(C_t) - \widetilde{V}(L_t) \right), \tag{97}$$

using

$$\widetilde{V}(L_t) = \frac{L_t^{1+\psi}}{1+\psi}. \tag{98}$$

We approximate the expected discounted utility in equation (97) around the detrended efficient equilibrium (allowing for shocks and imposing flexible prices). Using the aggregate growth trend $\Gamma^e_t = A_t Q_t / \Delta^e_t$ and the detrended version of the aggregate technology in equation (50), we rearrange equation (97) according to

$$
\begin{aligned}
W &= E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Big(\ln(c_t \Gamma_t^e) - \widetilde{V}(L_t) \Big) \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Big(\ln(c_t) - \widetilde{V}(L_t) \Big) + t.i.p. \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Bigg(\ln(y_t) - \widetilde{V}(y_t \frac{\Delta_t}{\Delta_t^e}) \Bigg) + t.i.p., \qquad (99)
\end{aligned}
$$

where $t.i.p.$ denotes terms that are independent of policy and the last equation uses market clearing $c_t = y_t$. We rearrange the definition of the output gap (19) according to

$$
\begin{aligned}
x_t &= \ln(Y_t/\Gamma_t^e) - \ln(Y_t^e/\Gamma_t^e) \\
&= \ln y_t - \ln y_t^e, \qquad (100)
\end{aligned}
$$

and define

$$
\widehat{\Delta}_t \equiv \ln \Delta_t - \ln \Delta_t^e. \qquad (101)
$$

Using equations (100) and (101), we further rearrange equation (99) according to

$$
\begin{aligned}
W &= E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Big(\ln(y_t^e \cdot e^{x_t}) - \widetilde{V}(y_t^e \cdot e^{x_t + \widehat{\Delta}_t}) \Big) + t.i.p. \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Big(\ln(e^{x_t}) - \widetilde{V}(y_t^e \cdot e^{x_t + \widehat{\Delta}_t}) \Big) + t.i.p. \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \underbrace{\Big(x_t - \widetilde{V}(y_t^e \cdot e^{x_t + \widehat{\Delta}_t}) \Big)}_{\equiv \widetilde{U}_t} + t.i.p. \qquad (102)
\end{aligned}
$$

We approximate the relevant part of the period utility function, denoted by $\widetilde{U}_t$, to second order around the efficient equilibrium ($\ln \Pi_t = \ln \Pi_t^\star, \ln y_t = \ln y_t^e$) in which $x_t = \widehat{\Delta}_t = 0$ and the condition $\frac{\theta}{\theta-1}\frac{1}{1+\tau} = 1$ holds.

The first derivative required for this approximation is given by

$$
\begin{aligned}
\frac{\partial \widetilde{U}_t}{\partial x_t}\bigg|_{x_t = \widehat{\Delta}_t = 0} &= 1 - \frac{\partial \widetilde{V}}{\partial L_t} y_t^e \cdot e^{x_t + \widehat{\Delta}_t}\bigg|_{x_t = \widehat{\Delta}_t = 0} = 1 - \frac{\partial \widetilde{V}}{\partial L_t}\bigg|_{x_t = \widehat{\Delta}_t = 0} y_t^e \\
&= 0. \qquad (103)
\end{aligned}
$$

The last equality uses that

$$\left.\frac{\partial\widetilde{V}}{\partial L_t}\right|_{x_t=\widehat{\Delta}_t=0} y_t^e = 1, \tag{104}$$

which follows from evaluating $\partial\widetilde{V}/\partial L_t = L_t^{\psi}$ at $L_t^e = 1$ and using $y_t^e = 1$ from equation (58). The second derivative required for approximating equation (102) is given by

$$\begin{aligned}
\left.\frac{\partial^2\widetilde{U}_t}{(\partial x_t)^2}\right|_{x_t=\widehat{\Delta}_t=0} &= -\frac{\partial^2\widetilde{V}}{(\partial L_t)^2}\left(y_t^e\cdot e^{x_t+\widehat{\Delta}_t}\right)^2 - \left.\frac{\partial\widetilde{V}}{\partial L_t}y_t^e\cdot e^{x_t+\widehat{\Delta}_t}\right|_{x_t=\widehat{\Delta}_t=0} \\
&= -\left(\left.\frac{\partial^2\widetilde{V}}{(\partial L_t)^2}\right|_{x_t=\widehat{\Delta}_t=0}(y_t^e)^2 + \left.\frac{\partial\widetilde{V}}{\partial L_t}\right|_{x_t=\widehat{\Delta}_t=0} y_t^e\right) \\
&= -\left(\left.\frac{\partial^2\widetilde{V}}{(\partial L_t)^2}\right|_{x_t=\widehat{\Delta}_t=0} + 1\right),
\end{aligned} \tag{105}$$

where the last equality again uses equation (104) and $y_t^e = 1$. The third derivative required for approximating equation (102) is given by

$$\begin{aligned}
\left.\frac{\partial\widetilde{U}_t}{\partial\widehat{\Delta}_t}\right|_{x_t=\widehat{\Delta}_t=0} &= -\left.\frac{\partial\widetilde{V}}{\partial L_t}y_t^e\cdot e^{x_t+\widehat{\Delta}_t}\right|_{x_t=\widehat{\Delta}_t=0} = -\left.\frac{\partial\widetilde{V}}{\partial L_t}\right|_{x_t=\widehat{\Delta}_t=0} y_t^e \\
&= -1,
\end{aligned} \tag{106}$$

where the last equality again uses equation (104).

Derivatives (103), (105) and (106) suffice to obtain a second-order expansion of $\widetilde{U}_t$ in equation (102) that involves all terms affected by government policy because as we show in the next section C.1, the productivity-adjusted price-dispersion gap is of second order, $\widehat{\Delta}_t \sim O(2)$, whenever its initial value is of second order, $\widehat{\Delta}_{-1} \sim O(2)$. By construction, $\widehat{\Delta}_t$ is at its maximum value at the point of approximation and hence all its first-order derivatives are zero at this point. Thus, $\widehat{\Delta}_t \sim O(2)$ necessarily holds. This result, which is well-known to arise in the standard New Keynesian model approximated around the efficient equilibrium, also emerges in the framework with heterogeneous firms considered here.[35] Therefore, we neither need to consider the second derivative of $\widetilde{U}_t$ with respect to $\widehat{\Delta}_t$ nor cross derivatives of $\widetilde{U}_t$ involving $\widehat{\Delta}_t$. We now show $\widehat{\Delta}_t \sim O(2)$ formally.

[35] Derivations for the standard New Keynesian model are nested in the derivations here and are obtained when imposing the parameter value $\delta = 0$ and $\Pi_t^\star = 1$.

C.1 Dispersion Gap is of Second Order

Substituting equation (65) into equation (64) yields

$$\frac{\Delta_t}{\Delta_t^e} = \left(1-\alpha(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}\right)\left(\frac{1-\alpha(1-\delta)(\Pi_t/\Xi_{t-1,t})^{\theta-1}}{1-\alpha(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}}\right)^{\frac{-\theta}{1-\theta}}$$
$$+\,\alpha(1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1}\left(\frac{\Pi_t/\Xi_{t-1,t}}{\Pi_t^\star/\Xi_{t-1,t}^\star}\right)^{\theta}\left(\frac{\Delta_{t-1}}{\Delta_{t-1}^e}\right).$$

Summarizing terms in the previous equation and using the definition

$$\rho_t \equiv (1-\delta)(\Pi_t^\star/\Xi_{t-1,t}^\star)^{\theta-1} \tag{107}$$

to rearrange it yields

$$\frac{\Delta_t}{\Delta_t^e} = (1-\alpha\rho_t)^{\frac{1}{1-\theta}}\left(1-\alpha\rho_t\left(\frac{\Pi_t/\Xi_{t-1,t}}{\Pi_t^\star/\Xi_{t-1,t}^\star}\right)^{\theta-1}\right)^{\frac{-\theta}{1-\theta}} + \alpha\rho_t\left(\frac{\Pi_t/\Xi_{t-1,t}}{\Pi_t^\star/\Xi_{t-1,t}^\star}\right)^{\theta}\left(\frac{\Delta_{t-1}}{\Delta_{t-1}^e}\right).$$

Defining

$$\widetilde{\Pi}_t \equiv \Pi_t/\Xi_{t-1,t} \tag{108}$$

$$\widetilde{\Pi}_t^\star \equiv \Pi_t^\star/\Xi_{t-1,t}^\star, \tag{109}$$

we obtain for the previous equation that

$$\frac{\Delta_t}{\Delta_t^e} = (1-\alpha\rho_t)^{\frac{1}{1-\theta}}\left(1-\alpha\rho_t\left(\frac{\widetilde{\Pi}_t}{\widetilde{\Pi}_t^\star}\right)^{\theta-1}\right)^{\frac{-\theta}{1-\theta}} + \alpha\rho_t\left(\frac{\widetilde{\Pi}_t}{\widetilde{\Pi}_t^\star}\right)^{\theta}\left(\frac{\Delta_{t-1}}{\Delta_{t-1}^e}\right).$$

Taking logs of this equation, expressing inflation variables in logs and using the definition of $\widehat{\Delta}_t$ in equation (101) yields

$$\widehat{\Delta}_t = \ln\left((1-\alpha\rho_t)^{\frac{1}{1-\theta}}\left(1-\alpha\rho_t e^{(\theta-1)\ln(\widetilde{\Pi}_t/\widetilde{\Pi}_t^\star)}\right)^{\frac{-\theta}{1-\theta}} + \alpha\rho_t e^{\theta\ln(\widetilde{\Pi}_t/\widetilde{\Pi}_t^\star)}e^{\widehat{\Delta}_{t-1}}\right). \tag{110}$$

Dividing definition (108) by definition (109), taking logs and using $\Xi_{t-1,t} = (\Pi_{t-1}/\Pi^\star)^\varphi$ and $\Xi_{t-1,t}^\star = (\Pi_{t-1}^\star/\Pi^\star)^\varphi$, we further obtain

$$\ln\widetilde{\Pi}_t = \ln\widetilde{\Pi}_t^\star + \ln(\Pi_t/\Pi_t^\star) - \varphi\ln(\Pi_{t-1}/\Pi_{t-1}^\star) \tag{111}$$

which is linear in logs and hence implies no cross terms between $\ln\Pi_t$ and $\ln\Pi_{t-1}$ in the derivations below.

We now approximate the RHS of the equation (110) in $\ln \Pi_t$, $\ln \Pi_{t-1}$ and $\widehat{\Delta}_{t-1}$ around the point $\ln \Pi_t = \ln \Pi_t^\star$, $\ln \Pi_{t-1} = \ln \Pi_{t-1}^\star$ and $\widehat{\Delta}_{t-1} = 0$ accurate to first order and using equation (111). The first derivative required for this approximation is

$$\frac{\partial \widehat{\Delta}_t}{\partial \ln \widetilde{\Pi}_t} \cdot \frac{\partial \ln \widetilde{\Pi}_t}{\partial \ln \Pi_t} = 0. \tag{112}$$

This derivative is equal to zero at the point of approximation because we have that

$$\begin{aligned}\frac{\partial \widehat{\Delta}_t}{\partial \ln \widetilde{\Pi}_t} &= e^{-\widehat{\Delta}_t} \cdot \Bigg((1-\alpha\rho_t)^{\frac{1}{1-\theta}} \; \theta \left(1 - \alpha\rho_t e^{(\theta-1)\ln(\widetilde{\Pi}_t/\widetilde{\Pi}_t^\star)}\right)^{\frac{\theta}{\theta-1}-1} \\ &\qquad (-\alpha)\rho_t e^{(\theta-1)\ln(\widetilde{\Pi}_t/\widetilde{\Pi}_t^\star)} + \theta\alpha\rho_t e^{\theta\ln(\widetilde{\Pi}_t/\widetilde{\Pi}_t^\star)} e^{\widehat{\Delta}_{t-1}} \Bigg) \\ &= 0,\end{aligned} \tag{113}$$

where the second equality follows from evaluating the derivative at the point of approximation. Furthermore, from equation (111) we have that

$$\frac{\partial \ln \widetilde{\Pi}_t}{\partial \ln \Pi_t} = 1. \tag{114}$$

The second derivative required for the approximation of equation (110) is given by

$$\frac{\partial \widehat{\Delta}_t}{\partial \ln \widetilde{\Pi}_t} \cdot \frac{\partial \ln \widetilde{\Pi}_t}{\partial \ln \Pi_{t-1}} = 0, \tag{115}$$

where this derivative again is equal to zero at the point of approximation because derivative (113) is equal to zero at this point and equation (111) implies that

$$\frac{\partial \ln \widetilde{\Pi}_t}{\partial \ln \Pi_{t-1}} = -\varphi. \tag{116}$$

The third derivative required for the approximation of equation (110) is given by

$$\begin{aligned}\frac{\partial \widehat{\Delta}_t}{\partial \widehat{\Delta}_{t-1}} &= e^{-\widehat{\Delta}_t} \alpha\rho_t e^{\theta\ln(\widetilde{\Pi}_t/\widetilde{\Pi}_t^\star)} e^{\widehat{\Delta}_{t-1}} \\ &= \alpha\rho_t,\end{aligned}$$

where the last equality follows from evaluating this derivative at the point of approximation. We thus obtain that

$$\widehat{\Delta}_t = \alpha\rho_t\widehat{\Delta}_{t-1} + O(2). \tag{117}$$

Using the definition (24),

$$\rho \equiv (1-\delta)(\Pi^\star)^{\theta-1},$$

we obtain $\rho_t = \rho + O(1)$ and thus can write

$$\widehat{\Delta}_t = \alpha\rho\widehat{\Delta}_{t-1} + O(2), \tag{118}$$

because the first-order terms only enter as product with other first-order terms and thus are absorbed by the residual $O(2)$. Therefore, if the initial value of the productivity-adjusted price-dispersion gap satisfies

$$\widehat{\Delta}_{-1} \sim O(2), \tag{119}$$

as we assume in Assumption 1 in the main text, it will be the case that $\widehat{\Delta}_t \sim O(2)$ for all $t \geq 0$.

C.2 Welfare as Function of Output and Dispersion Gaps

Using derivatives (103), (105) and (106) and equations (118) and (119), the approximation to equation (102) is given by

$$W = E_0\left[\sum_{t=0}^{\infty}\beta^t\xi_t\left(-\frac{1}{2}\left(\left.\frac{\partial^2\widetilde{V}}{(\partial L_t)^2}\right|_{x_t=\widehat{\Delta}_t=0} + 1\right)x_t^2 - \widehat{\Delta}_t\right)\right] + t.i.p. + O(3)\ . \tag{120}$$

Since $\xi_t = 1 + O(1)$ and

$$\left.\frac{\partial^2\widetilde{V}}{(\partial L_t)^2}\right|_{x_t=\widehat{\Delta}_t=0} = \psi\left(L_t^e\right)^{\psi-1} = \psi,$$

which follows from equation (98) and $L_t^e = 1$ in equation (58), we can rearrange equation (120) to obtain

$$W = -E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{1}{2}(1+\psi)x_t^2 + \widehat{\Delta}_t\right) + t.i.p. + O(3). \tag{121}$$

C.3 Dispersion Gap as Function of Inflation Gap

We now approximate equation (110) to express $\widehat{\Delta}_t$ to second-order accuracy as a function of current and lagged inflation rates, $\ln \Pi_t$ and $\ln \Pi_{t-1}$. We can rearrange the second derivative of $\widehat{\Delta}_t$ with respect to the current inflation rate according to

$$\frac{\partial}{\partial \ln \Pi_t}\left(\frac{\partial \widehat{\Delta}_t}{\partial \ln \Pi_t}\right) = \frac{\partial}{\partial \ln \widetilde{\Pi}_t}\frac{\partial \ln \widetilde{\Pi}_t}{\partial \ln \Pi_t}\left(\frac{\partial \widehat{\Delta}_t}{\partial \ln \widetilde{\Pi}_t}\frac{\partial \ln \widetilde{\Pi}_t}{\partial \ln \Pi_t}\right) = \frac{\partial^2 \widehat{\Delta}_t}{(\partial \ln \widetilde{\Pi}_t)^2},$$

where the second equality follows from using equation (114). Along similar lines, the second derivative of $\widehat{\Delta}_t$ with respect to the lagged inflation rate can be rearranged according to

$$\frac{\partial}{\partial \ln \Pi_{t-1}}\left(\frac{\partial \widehat{\Delta}_t}{\partial \ln \Pi_{t-1}}\right) = \frac{\partial}{\partial \ln \widetilde{\Pi}_t}\frac{\partial \ln \widetilde{\Pi}_t}{\partial \ln \Pi_{t-1}}\left(\frac{\partial \widehat{\Delta}_t}{\partial \ln \widetilde{\Pi}_t}\frac{\partial \ln \widetilde{\Pi}_t}{\partial \ln \Pi_{t-1}}\right) = \frac{\partial^2 \widehat{\Delta}_t}{(\partial \ln \widetilde{\Pi}_t)^2}\cdot(-\varphi)^2,$$

where the second equality follows from using equation (116). The cross derivative of $\widehat{\Delta}_t$ with respect to current and lagged inflation rates can be rearranged according to

$$\frac{\partial}{\partial \ln \Pi_{t-1}}\left(\frac{\partial \widehat{\Delta}_t}{\partial \ln \Pi_t}\right) = \frac{\partial}{\partial \ln \widetilde{\Pi}_t}\frac{\partial \ln \widetilde{\Pi}_t}{\partial \ln \Pi_{t-1}}\left(\frac{\partial \widehat{\Delta}_t}{\partial \ln \widetilde{\Pi}_t}\frac{\partial \ln \widetilde{\Pi}_t}{\partial \ln \Pi_t}\right) = \frac{\partial^2 \widehat{\Delta}_t}{(\partial \ln \widetilde{\Pi}_t)^2}\cdot(-\varphi),$$

where the second equality follows from using equations (114) and (116). Taking the derivative of equation (113) with respect to the inflation variable yields

$$\begin{aligned}\frac{\partial^2 \widehat{\Delta}_t}{(\partial \ln \widetilde{\Pi}_t)^2} =& e^{-\widehat{\Delta}_t}\Bigg((1-\alpha\rho_t)^{\frac{1}{1-\theta}}\theta\left(1-\alpha\rho_t e^{(\theta-1)\ln(\widetilde{\Pi}_t/\widetilde{\Pi}_t^\star)}\right)^{\frac{\theta}{\theta-1}-2}(\alpha\rho_t)^2 e^{2(\theta-1)\ln(\widetilde{\Pi}_t/\widetilde{\Pi}_t^\star)} \\ &+ (1-\alpha\rho_t)^{\frac{1}{1-\theta}}\ \theta\left(1-\alpha\rho_t e^{(\theta-1)\ln(\widetilde{\Pi}_t/\widetilde{\Pi}_t^\star)}\right)^{\frac{\theta}{\theta-1}-1}(\theta-1)(-\alpha)\rho_t e^{(\theta-1)\ln(\widetilde{\Pi}_t/\widetilde{\Pi}_t^\star)} \\ &+ \theta^2\alpha\rho_t e^{\theta\ln(\widetilde{\Pi}_t/\widetilde{\Pi}_t^\star)}e^{\widehat{\Delta}_{t-1}}\Bigg),\end{aligned}$$

and evaluating this derivative at $\ln \widetilde{\Pi}_t = \ln \widetilde{\Pi}_t^\star$ and $\widehat{\Delta}_t = \widehat{\Delta}_{t-1} = 0$ and simplifying terms yields

$$\begin{aligned}\frac{\partial^2 \widehat{\Delta}_t}{(\partial \ln \widetilde{\Pi}_t)^2} &= \theta\frac{(\alpha\rho_t)^2}{1-\alpha\rho_t} - \theta(\theta-1)\alpha\rho_t + \theta^2\alpha\rho_t \\ &= \theta\left(\frac{\alpha\rho_t}{1-\alpha\rho_t}\right).\end{aligned}$$

Using equation (117), the second-order approximation to equation (110) is then given by

$$\widehat{\Delta}_t = \alpha\rho_t \cdot \widehat{\Delta}_{t-1} \tag{122}$$
$$+ \frac{1}{2}\theta\left(\frac{\alpha\rho_t}{1-\alpha\rho_t}\right)\cdot\left(\left(\ln\frac{\Pi_t}{\Pi_t^\star}\right)^2 - 2\left(\ln\frac{\Pi_t}{\Pi_t^\star}\right)\left(\varphi\ln\frac{\Pi_{t-1}}{\Pi_{t-1}^\star}\right) + \left(\varphi\ln\frac{\Pi_{t-1}}{\Pi_{t-1}^\star}\right)^2\right) + O(3).$$

From definitions (16) and (17), we obtain that

$$\ln\frac{\Pi_t}{\Pi_t^\star} = \ln\frac{\Pi_t/\Pi^\star}{\Pi_t^\star/\Pi^\star} = \pi_t - \pi_t^\star,$$

and substituting this into equation (122) and completing the squares yields

$$\widehat{\Delta}_t = \alpha\rho_t \cdot \widehat{\Delta}_{t-1} + \frac{1}{2}\theta\left(\frac{\alpha\rho_t}{1-\alpha\rho_t}\right)\cdot\left(\pi_t - \pi_t^\star - \varphi(\pi_{t-1} - \pi_{t-1}^\star)\right)^2 + O(3), \tag{123}$$

since all further terms are absorbed into the approximation residual given that $\widehat{\Delta}_t \sim O(2)$, as is shown in section C.1. Again using that $\rho_t = \rho + O(1)$ and hence

$$\frac{\alpha\rho_t}{1-\alpha\rho_t} = \frac{\alpha\rho}{1-\alpha\rho} + O(1),$$

we obtain for the second-order approximation in equation (123) that

$$\widehat{\Delta}_t = \alpha\rho \cdot \widehat{\Delta}_{t-1} + \frac{1}{2}\theta\left(\frac{\alpha\rho}{1-\alpha\rho}\right)\cdot\left(\pi_t - \pi_t^\star - \varphi(\pi_{t-1} - \pi_{t-1}^\star)\right)^2 + O(3), \tag{124}$$

because first-order terms only enter as products with second-order terms and thus are also absorbed into the residual $O(3)$. In the special case with $\delta = 0$, $\Pi_t^\star = \Pi^\star = 1$ and hence $\rho = 1$, equation (124) reduces to the equation that describes the evolution of price dispersion in the standard New Keynesian model with price indexation to lagged inflation. Summing equation (124) over time with discounting, and rearranging the result, yields

$$\sum_{t=0}^{\infty}\beta^t\widehat{\Delta}_t = \alpha\rho\beta\sum_{t=0}^{\infty}\beta^t\widehat{\Delta}_t + \frac{1}{2}\theta\left(\frac{\alpha\rho}{1-\alpha\rho}\right)\sum_{t=0}^{\infty}\beta^t\left(\pi_t - \pi_t^\star - \varphi(\pi_{t-1} - \pi_{t-1}^\star)\right)^2$$
$$+ O(3) + t.i.p.$$

after absorbing the initial value $\widehat{\Delta}_{-1}$ into terms independent of policy. Rearranging the previous equation yields

$$\sum_{t=0}^{\infty}\beta^t\widehat{\Delta}_t = \frac{1}{2}\theta\left(\frac{\alpha\rho}{(1-\alpha\rho)(1-\alpha\rho\beta)}\right)\sum_{t=0}^{\infty}\beta^t\left(\pi_t-\pi_t^\star-\varphi(\pi_{t-1}-\pi_{t-1}^\star)\right)^2 + O(3)+t.i.p. \tag{125}$$

C.4 Welfare as Function of Output and Inflation Gaps

Substituting equation (125) into equation (121) yields a second-order approximation to welfare expressed in terms of the output gap and the quasi-differenced inflation gap,

$$W = -\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{1+\psi}{\theta}\frac{(1-\alpha\rho)(1-\alpha\rho\beta)}{\alpha\rho}x_t^2+\left(\pi_t-\pi_t^\star-\varphi(\pi_{t-1}-\pi_{t-1}^\star)\right)^2\right) + O(3)+t.i.p. \tag{126}$$

with $\rho=(1-\delta)(\Pi^\star)^{\theta-1}$. The weight on the quasi-differenced inflation gap is normalized to unity, and the weight on the output gap is equal to the slope of the Phillips curve divided by θ. The representation of approximate welfare in Proposition 1 corresponds to the case without price indexation with $\varphi=0$.

D Proof of Lemma 2

Since the natural real rate does not move following a monetary policy shock and the real interest rate in equilibrium is given by

$$i_t - E_t\pi_{t+1} = -\rho^{t+1}(1-\rho)\widehat{g}_0,$$

the Euler equation (27) implies

$$x_t = E_t x_{t+1} + \rho^{t+1}(1-\rho)\widehat{g}_0.$$

Forward iteration yields

$$x_t = \rho^{t+1}\widehat{g}_0,$$

and using equation (37) to substitute for $\widehat{g}_0$ yields

$$x_t = -\frac{1}{1-\rho} r_t^n.$$

Since the optimal inflation rate does not respond following a monetary policy shock, the Phillips curve (26) implies

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$

Forward iteration and using $x_t = \rho^{t+1} \widehat{g}_0$ yields

$$\pi_t = \frac{\kappa}{1-\beta\rho} \rho^{t+1} \widehat{g}_0,$$

an using equation (37) to substitute for $\widehat{g}_0$ yields

$$\pi_t = -\frac{\kappa}{(1-\rho)(1-\beta\rho)} r_t^n.$$

Finally, since the efficient level of output does not respond to the monetary policy shock, we have $x_t = \widetilde{y}_t$.

E Proof of Lemma 3

Assuming $\hat{\imath}_t = 0$ in equilibrium yields for the Euler equation (27) that

$$x_t = E_t x_{t+1} + E_t \pi_{t+1} + r_t^n. \tag{127}$$

We require that $x_t = E_t x_{t+1} + \rho^{t+1}(1-\rho)\widehat{g}_0$ to obtain the same solution as in lemma 2, which jointly with equation (127) yields

$$r_t^n = \rho^{t+1}(1-\rho)\widehat{g}_0 - E_t \pi_{t+1}. \tag{128}$$

The solution for inflation in lemma 2 implies that

$$\begin{aligned} E_t \pi_{t+1} &= \pi_{t+1} \\ &= \frac{\kappa}{1-\beta\rho} \rho^{t+2} \widehat{g}_0. \end{aligned}$$

Substituting this equation into equation (128) yields

$$r_t^n = \rho^{t+1}(1-\rho)\widehat{g}_0 - \frac{\kappa}{1-\beta\rho}\rho^{t+2}\widehat{g}_0, \tag{129}$$

as stated in lemma 3.

F Cohort-Level Expenditure Shares

This appendix derives the evolution of cohort-level expenditure shares in response to the various (non-idiosyncratic) shocks in the model. Derivations in this appendix abstract from price indexation by imposing $\Xi_{t-k,t} = 1$ throughout.

Consider the expenditure share of a cohort with age s. Let $P^\star_{t-s,t-k}$ denote the optimal price of a firm that last experienced a δ-shock in $t-s$ and that has last reset its price in $t-k$, with $s \geq k \geq 0$. Let $m_t(s)$ denote the weighted average expenditure share in period t of the cohort of firms that last experienced a δ-shock in period $t-s$. For $s > k \geq 0$, this expenditure share is given by

$$m_t(s) = (1-\alpha)\sum_{k=0}^{s-1}\alpha^k (P^\star_{t-s,t-k}/P_t)^{1-\theta} + \alpha^s (P^\star_{t-s,t-s}/P_t)^{1-\theta}, \tag{130}$$

comprising α^s firms that have not had a chance to optimally reset prices since receiving the δ-shock and $(1-\alpha)\alpha^k$ firms that have last adjusted prices $k < s$ periods ago. For $s = 0$, the corresponding expenditure share is given by $m_t(0) = (P^\star_{t,t}/P_t)^{1-\theta}$. From equation (7), it follows that one can express the weighted sum of cohort expenditure shares according to

$$1 = \sum_{s=0}^{\infty}(1-\delta)^s \delta\, m_t(s), \tag{131}$$

where δ denotes the mass of firms that experience a δ-shock each period and $(1-\delta)^s$ denotes the share of those firms that have not undergone another δ-shock for s periods.

We rearrange equation (130) to obtain a tractable representation of the cohort expenditure share. Consider the optimal price $P^\star_{t-s,t}$ of a firm that sustained a δ-shock $s > 0$ periods ago, but can adjust the price in t due to the occurrence of a Calvo shock. Also, consider the price $P^\star_{t,t}$ of a firm where a δ-shock occurs in period t. The optimal price setting equation (11) then implies

$$P^\star_{t,t} = P^\star_{t-s,t}\left(\frac{g_t \times \cdots \times g_{t-s+1}}{q_t \times \cdots \times q_{t-s+1}}\right). \tag{132}$$

Using equation (132) to substitute prices $P^\star_{t-s,t-k}$ by prices $P^\star_{t-k,t-k}$ in equation (130) and using notation $p^\star_{t-k} = P^\star_{t-k,t-k}/P_{t-k}$ yields

$$\begin{aligned} m_t(s) &= (1-\alpha)\sum_{k=0}^{s-1}\alpha^k\left(p^\star_{t-k}\left(\Pi_t \times \cdots \Pi_{t-k+1}\right)^{-1}\left(\frac{g_{t-k}\times\cdots\times g_{t-s+1}}{q_{t-k}\times\cdots\times q_{t-s+1}}\right)^{-1}\right)^{1-\theta} \\ &+ \alpha^s\left(p^\star_{t-s}\left(\Pi_t \times \cdots \Pi_{t-s+1}\right)^{-1}\right)^{1-\theta}, \end{aligned} \tag{133}$$

where we obtain $m_t(0) = (p^\star_t)^{1-\theta}$ for age $s = 0$. We linearize the expenditure share at its efficient steady state in which $\Pi = \Pi^\star = g/q$. In this steady state, equation (133) implies that

$$m^e(s) = (p^e)^{1-\theta}(g/q)^{(\theta-1)\cdot s}. \tag{134}$$

Linearizing equation (133) at this steady state yields

$$\begin{aligned} \widehat{m}_t(s) = (1-\theta)\Bigg((1-\alpha)&\sum_{k=0}^{s-1}\alpha^k\breve{p}^\star_{t-k} + \alpha^s\breve{p}^\star_{t-s} \\ -\sum_{k=0}^{s-1}(1-\alpha^{k+1})\left(\widehat{g}_{t-k}-\widehat{q}_{t-k}\right) &- \sum_{k=0}^{s-1}\alpha^{k+1}\pi_{t-k}\Bigg), \end{aligned} \tag{135}$$

where we denote the percentage deviation of the relative reset price from its value in the efficient steady state as $\breve{p}^\star_t = \ln p^\star_t - \ln p^e$.[36] To obtain a solution for this variable, we rearrange equation (45) imposing that $\Xi_{t-1,t} = 1$ and exploiting equation (56) in steady state. This yields

$$1 = \left[\alpha(1-\rho) + (1-\alpha)(\Delta^e_t/\Delta^e)^{1-\theta}\right](p^\star_t/p^e)^{1-\theta} + \alpha\rho\left(\Pi_t/\Pi^\star\right)^{\theta-1}. \tag{136}$$

Linearizing this equation at the efficient steady state, at which $\Delta^e_t = \Delta^e$, $p^\star_t = p^e$ and $\Pi = \Pi^\star = g/q$, yields

$$(1-\alpha\rho)\breve{p}^\star_t = \alpha\rho\pi_t + \frac{(1-\alpha)\rho}{1-\rho}\pi^\star_t. \tag{137}$$

For the impulse response functions (IRF), we use equations (135) and (137) jointly with

[36] This variable differs from the relative price gap used before, $\widehat{p}^\star_t = \ln p^\star_t - \ln p^e_t$, which features an additional time subscript.

the remaining approximate equilibrium conditions to compute $\widehat{m}_{t_0+s}(s)$ as a function of cohort age s assuming the shock hits at time t_0. The coefficients of this IRF are cohort-age dependent since we linearize at the efficient market share which evolves with cohort age s.