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Multiple Equilibria? Don't Panic! – A Hitchhiker's Guide to Global Games*

– Prepared for the Handbook of Macprudential Regulation –

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Abstract

This article provides a practical overview for applying the global games approach to solve models with multiple equilibria that are often used in discussions on financial and macroprudential policies. Global games offer a tractable approach to resolve multiple equilibria by introducing incomplete information, thereby yielding unique equilibrium predictions. The article proceeds along the lines of a simple regime change game with strategic complementarities. Starting from the canonical regime change game with homogeneous players, it extends the discussion to include heterogeneous groups of players and interlinkages across different institutions with different sets of players. These extensions highlight not only how strategic complementarities can amplify fragility across players and institutions but also how heterogeneity and interlinkages affects the design of micro- and macroprudential policy interventions. Finally, the article briefly discusses the application of global games to dynamic coordination games.

Keywords: Global Games, Multiple Equilibria, Coordination Games.

JEL Classification: C72, D82, G01.

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1 Introduction

A large number of models in economics and finance exhibit multiple self-fulfilling equilibria, including those that study financial and banking crises and frequently feature in micro- and macroprudential policy discussions. For instance, the seminal [Diamond and Dybvig \(1983\)](#) banking model exhibits a run equilibrium, where depositors panic and withdraw their bank deposits, and another equilibrium where no bank-run occurs. Similarly, the currency peg model by [Obstfeld \(1996\)](#) features an equilibrium where a speculative attack leads to the abandonment of a fixed exchange rate regime and another one where no attack occurs and the exchange rate peg can be sustained. And in [Calvo \(1988\)](#), an equilibrium where a sovereign borrower is unable to pay its debt exists alongside an equilibrium where no repudiation occurs and the sovereign repays its debt.

The equilibria in these models are self-fulfilling, i.e., agents, expecting a future outcome, adopt a behaviour that, in aggregate, gives rise to the previously expected outcome. The agents' initially held beliefs are therefore logically consistent because they are vindicated by the results that follow. The presence of multiple self-fulfilling equilibria implies that there exist several distinct sets of logically consistent beliefs for a given set of fundamental model parameters. Thus, agents' beliefs are left indeterminate by the fundamentals of the model ([Morris and Shin, 2000](#)). What is more, the different equilibria can often be Pareto-ranked, i.e., they can be associated with markedly different welfare levels ([Cooper and John, 1988](#)). Once the economy gets stuck in the Pareto-inferior equilibrium, we say that coordination failure occurs.

Equilibrium multiplicity offers both advantages and disadvantages. The ability to rank equilibria, for example, provides an intuitive way of formalizing the notion of the fragility of a particular economy or institution as the possibility of suddenly slipping – by a mere change in beliefs – into a Pareto-inferior equilibrium. At the same time,

however, the welfare outcomes in multiple equilibrium models do not correlate with the underlying fundamentals. This property contrasts with the notion that inferior outcomes should be observed more often when fundamentals are worse ([Morris and Shin, 2000](#)). Finally, as fundamentals do not uniquely pin down the equilibrium, clear-cut predictions about the outcome that eventually prevails are not attainable, thereby inhibiting policy analyses by means of standard comparative statics analysis ([Morris and Shin, 2000, 2003](#)).

Global games provide a tractable approach to addressing the problems associated with equilibrium multiplicity without giving up the advantages. The idea underlying this approach is to abandon the assumption that players have common knowledge about the game that is being played and about the eventual play in the game (complete information). [Morris and Shin \(2000\)](#) emphasize that it is this assumption that gives rise to equilibrium multiplicity because it leads players' beliefs and actions to be perfectly coordinated in equilibrium.

The global games approach instead embeds the original game with multiple equilibria into a game of incomplete information. Players do not know the actual game, which is drawn randomly from a class of possible games that differ with respect to a payoff-relevant parameter. Players obtain private information about the payoff-relevant parameter and use it to form beliefs about the game they are playing. Hence, in a 'global game', players' strategies take into account all situations that could arise in all possible games ([Heinemann, 2005](#)). When the class of games is sufficiently large and includes games with different equilibrium structures, a unique equilibrium can be obtained where players switch actions when they infer from their private information that the payoff-relevant parameter is sufficiently large ([Carlsson and van Damme, 1993](#)).¹

¹[Heinemann et al. \(2004\)](#) provide experimental evidence for the predictions of the theory of global games. In their experimental setting, subjects play according to threshold strategies and variations in their settings corroborate the theoretical comparative statics predictions.

The present chapter offers a ‘hitchhiker’s guide’ to applying the theory of global games for questions concerning micro- and macroprudential regulation. Our goal is to provide practitioners with a primer on how to apply global game models to prudential policy analysis. Throughout this chapter, we focus on a rather simple type of game, called a static *regime change game*.² In a typical regime change game, players decide on either attacking a status quo regime or abstaining from attack. If sufficiently many players attack, the status quo is abandoned, leading to a discrete jump in players’ payoffs. Under complete information, the regime change game exhibits two pure-strategy equilibria, making it the ideal candidate game to study the application of the global games approach.

Since [Carlsson and van Damme \(1993\)](#) introduced the global games approach, the epistemological underpinnings of global games have developed substantially.³ While much of this material is beyond the scope of this introductory chapter, we briefly highlight some of the most important papers as a reference for the interested reader. In particular, [Frankel et al. \(2003\)](#) show that the global games technique extends to games of strategic complementarities with an arbitrary number of players and actions. [Basteck et al. \(2013\)](#) show how the equilibrium selection in games with more than two actions may be obtained by decomposing the original many-action game into a series of simpler binary-action games. Moreover, recent research has focused on understanding why global games deliver unique equilibrium predictions. [Morris et al. \(2016\)](#) argue that the result crucially rests on the notion of uniform rank beliefs, i.e., each player believes that it is equally likely that all others players have higher (or lower) types. And recently, [Morris and Yildiz \(2019\)](#) show that uniform rank beliefs arise following a large shock with a common component drawn from a fat-tailed distribution.⁴

²Regime change games have been used to study currency attacks ([Morris and Shin, 1998](#); [Corsetti et al., 2004](#)), political revolutions ([Edmond, 2013](#)) and also bank runs and financial fragility ([Rochet and Vives, 2004](#); [Goldstein and Pauzner, 2005](#)).

³[Morris and Shin \(2004\)](#) and [Angeletos and Lian \(2016\)](#) provide deep-dives into the theory and applications of global games.

⁴Other seminal papers on the foundations of games with strategic complementarities include [Topkis \(1979\)](#); [Vives \(1990\)](#); [Milgrom and Roberts \(1990\)](#) and [Van Zandt and Vives \(2007\)](#).

The remainder of the chapter is structured as follows. [Section 2](#) presents the canonical regime change model with a continuum of identical and atomistic players and derives the global game solution. This benchmark model serves as a way to formalizing the notion of fragility of financial institutions and we discuss some of its applications in the bank-run literature in [Section 2.4](#). [Section 3](#) extends the canonical model to include a single large player as well as groups of players who differ in their payoffs. The models in this section allow to gear the canonical model towards applications in financial markets where large institutional investors or different creditor groups of financial institutions interact. To further emphasize macroprudential issues, [Section 4](#) considers the role of interlinkages between different institutions by considering simultaneous interdependent regime change models. Finally, [Section 5](#) provides a short literature overview on dynamic global games. A final section concludes. Most of the derivations of the results in the main body of the text can be found in [Appendix A](#). In [Appendix B](#) we provide a brief discussion of the popular Gaussian version of the regime change model that is also used extensively in the analysis of information and disclosure policies.

2 The Canonical Regime Change Game

2.1 Description of the Basic Model

There are two possible regimes, the status quo and an alternative. There is a continuum of players, indexed by $i \in I \subset \mathbf{R}_+$. For simplicity, we assume $I \equiv [0, 1]$. Players choose simultaneously to either attack the status quo or refraining from attack. Specifically, we denote the action of a typical player i by $a_i \in \{0, 1\}$ where 1 denotes attacking and 0 refraining from attacking. The aggregate action is

$$a \equiv \int_{i \in I} a_i.$$

The status quo is in place as long as

$$R(a, \theta) \geq 0,$$

where $\theta \in \Theta \subset \mathbf{R}$ is called the *state* or *fundamental* and the function $R : I \times \Theta \rightarrow \mathbf{R}$ is assumed to be continuous and differentiable in both arguments.

Table 1 presents the payoff to player i from playing either $a_i = 1$ or $a_i = 0$, which depends on the outcome of the regime and where we assume $B > C$:

	$R(a, \theta) < 0$	$R(a, \theta) \geq 0$
$a_i = 1$	$B - C$	$B - C$
$a_i = 0$	0	B

Table 1: Payoff matrix

The payoff difference that measures the differential payoff from ‘attack’ over ‘re-frain’ for a given regime outcome is given by:

$$\pi_i(a, \theta) = \begin{cases} B - C & \text{if } R(a, \theta) < 0 \\ -C & \text{if } R(a, \theta) \geq 0. \end{cases} \quad (1)$$

A player’s best reply to aggregate action a is $a_i = 1$ if and only if $\pi_i(a, \theta) > 0 \Leftrightarrow R(a, \theta) < 0$.

Assumption 1. *The payoff difference $\pi_i(a, \theta)$ satisfies for each $i \in I$:*

1. **(Global) Strategic complementarities.** $R(a, \theta)$ strictly decreases in a .
2. **State monotonicity.** $R(a, \theta)$ increases in θ .
3. **Dominance bounds.** There exist $\underline{\theta} \in \Theta$ and $\bar{\theta} \in \Theta$ such that $R(0, \underline{\theta}) = 0 = R(I, \bar{\theta})$.

The assumption of *global strategic complementarities* is tantamount to saying that players always have an incentive to do what others are doing. *State monotonicity* means

that the status quo is weaker for lower states. The assumption of *dominance bounds* implies the existence of sufficiently low (high) states where attack (refrain) becomes strictly dominant. From the perspective of a typical player, the behavior of other players becomes irrelevant for her action choice at these extreme values of θ .

We focus on symmetric pure strategy Nash equilibria. The aggregate action a^* constitutes a pure-strategy Nash equilibrium if each player's equilibrium action a_i^* constitutes a best response to the aggregate action a^* . An immediate implication of the existence of dominance bounds is that there exist well-defined subsets of Θ (which can be arbitrarily small) where a unique pure strategy Nash equilibrium obtains.

We use a linear specification which allows us to obtain closed-form solutions.

Assumption 2. *The function $R : I \times \Theta \rightarrow \mathbf{R}$ is given by*

$$R(a, \theta) = \tau\theta - \xi a - \delta, \quad \text{where } \tau, \xi, \delta > 0. \quad (2)$$

2.2 Complete Information and Multiple Equilibria

Under complete information players share common knowledge about the payoff functions and the game that is being played.

Proposition 1 (Complete information and multiple equilibria). *Under complete information, the action profile $(a_i^* = 0)_{i \in I}$ constitutes a (pure strategy) Nash equilibrium of the regime change game if $\theta \geq \frac{\delta}{\tau} \equiv \underline{\theta}$; the action profile $(a_i^* = 1)_{i \in I}$ constitutes a (pure strategy) Nash equilibrium if $\theta \leq \underline{\theta} \left(1 + \frac{\xi}{\delta}\right) \equiv \bar{\theta}$. There are multiple (pure strategy) Nash equilibria if $\theta \in [\underline{\theta}, \bar{\theta}]$.*

Equilibrium multiplicity is illustrated in [Fig. 1](#). For extreme values of θ , the equilibria are unique. At these states, the aggregate action plays no role for the behavior of a typical player because the status quo is either abandoned for sure ($\theta < \underline{\theta}$) or never cracks ($\theta > \bar{\theta}$). Insofar that $\xi > 0$, the presence of strategic complementarities implies

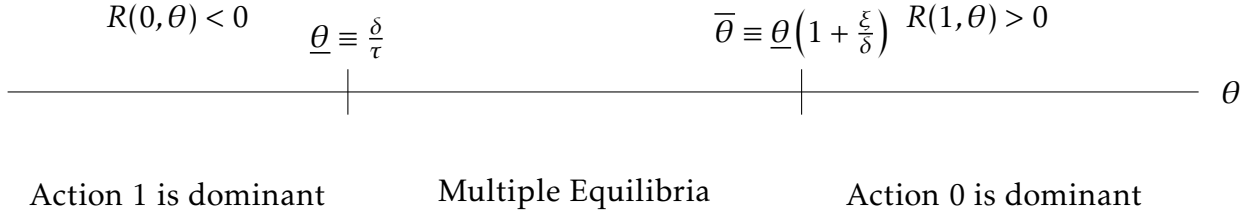


Figure 1: Tri-partite classification of the fundamental state according to equilibria.

that the interval $[\underline{\theta}, \bar{\theta}]$ is non-empty. For $\theta \in [\underline{\theta}, \bar{\theta}]$, the collapse of the status quo depends on the aggregate action and equilibrium multiplicity obtains. If players expect the status quo to fail because the aggregate attack is large, their best reply is to attack. If all players share this belief, they all attack which forces a regime change and validates the initially held belief. Conversely, if players believe that the aggregate attack is insufficient, their best reply is to refrain. If all players share this belief, the aggregate attack cannot enforce a regime change, thereby again vindicating the initial belief.

The equilibria in [Proposition 1](#) are Pareto-ranked. Given the payoffs in [Table 1](#), if $\theta \in (\underline{\theta}, \bar{\theta})$, the equilibrium where the status quo is maintained Pareto-dominates the equilibrium where the status quo is abandoned. This can be seen from the specification of the payoffs in [Table 1](#) where players incur an additional cost C when they attack.⁵

2.3 The Global Game Solution.

Unique threshold equilibrium. To select a unique equilibrium, we turn the complete information game into a game of incomplete information. Suppose that θ is a random variable drawn by Nature at the start of the game from a uniform distribution over Θ .⁶ Players do not know the realization of θ , but instead each player obtains a

⁵We could have specified the payoffs so as to make the ‘attack’-equilibrium Pareto-dominant. When applying the model to questions of financial fragility, it is more natural to assume that the survival of the status quo (e.g., the survival of a financial institution) is the Pareto-dominant (or efficient) outcome.

⁶The assumption of a uniform prior and uniformly distributed noise is for expositional simplicity. The application of the global game selection to games with global strategic complementarities is not confined to these specific assumptions. Rather, it applies for any continuously differentiable, strictly positive prior distribution and any integrable noise density ([Morris and Shin, 2003](#)). For example, an-

private signal

$$x_i = \theta + \varepsilon_i,$$

where $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$ and ε_i and ε_j are independent for $i \neq j$. Given her signal, each player i has the posterior belief

$$\theta|_{x_i} \sim U[x_i - \varepsilon, x_i + \varepsilon].$$

In this game of incomplete information, a strategy s_i for player i is a complete contingent plan that maps her signal into the action, i.e., $s_i : x_i \mapsto a_i$. We restrict attention to symmetric *monotone* or *threshold strategies*⁷ that can be expressed as a common critical threshold x^* such that for all $i \in [0, 1]$

$$s_i = \begin{cases} 1 & \text{if } x_i \leq x^* \\ 0 & \text{if } x_i > x^* \end{cases}$$

Given the symmetric monotone strategy around x^* , the share of players who choose action 1 is equal to the share of players with a signal below x^* . By the law of large numbers this share is just equal to the probability that a single player obtains a signal

other often-used alternative specification is the Gaussian model which we discuss in [Appendix B](#). The qualifier that strategic complementarities are *global* is important, however. Among others, [Goldstein and Pauzner \(2005\)](#), [Karp et al. \(2007\)](#) or [Daniels et al. \(2013\)](#) show that congestion effects can lead to *partial strategic complementarities*, i.e., the payoff difference increases for low values of the aggregate action, but decreases if the aggregate action becomes sufficiently large. Further assumptions on either the payoff difference or the noise distribution are then needed to derive a unique equilibrium. [Goldstein and Pauzner \(2005\)](#), for example, show that a unique equilibrium in monotone strategies can also be obtained with partial strategic complementarities. Their uniqueness proof relies on the payoff difference satisfying a single-crossing property and on the assumption of a uniform noise distribution. [Morris and Shin \(2003\)](#) provide a detailed discussion of the assumptions required to establish equilibrium uniqueness.

⁷The restriction to threshold strategies is without loss of generality. The threshold strategy is the unique strategy that survives iterated deletion of dominated strategies. We provide a proof in [Appendix A](#).

below the threshold. Thus, for a given θ , we can calculate the aggregate action as

$$a(x^*, \theta) = \mathbf{P}(x_i \leq x^* | \theta) = \begin{cases} 1 & \text{if } \theta < x^* - \varepsilon \\ \frac{x^* - \theta + \varepsilon}{2\varepsilon} & \text{if } \theta \in [x^* - \varepsilon, x^* + \varepsilon] \\ 0 & \text{if } \theta > x^* + \varepsilon \end{cases}$$

The equilibrium in this game can be summarized by the threshold x^* and a critical value θ^* such that $R(a(x^*, \theta), \theta) < 0$ if and only if $\theta < \theta^*$. In particular, given θ^* , the threshold x^* must be such that a player who observes $x_i = x^*$ is just indifferent between both of her actions. That is, x^* solves the *indifference condition*

$$\mathbf{P}(\theta \leq \theta^* | x^*) - \nu = 0, \quad (\text{IC})$$

where $\nu \equiv \frac{C}{B}$. Conversely, given the joint threshold strategy around x^* , the critical value θ^* , solves the *critical mass condition*

$$R(a(x^*, \theta^*), \theta^*) = 0 \quad \Leftrightarrow \quad \tau\theta^* - \delta - \xi a(x^*, \theta^*) = 0. \quad (\text{CM})$$

Under the present specification of the payoff difference and the assumption of uniform distributions, the solution to (IC) and (CM) is fairly straightforward by noticing that⁸

$$a(x^*, \theta^*) = \mathbf{P}(x_i \leq x^* | \theta^*) = 1 - \mathbf{P}(\theta \leq \theta^* | x^*) = 1 - \nu,$$

where the last equality follows from (IC). By substituting into (CM) one can solve for θ^* . Substituting θ^* back into (IC), one can solve for the threshold x^* . Importantly, it turns out that the monotone equilibrium of the game, summarized by the tuple (x^*, θ^*) , is indeed the unique equilibrium of the game. There are no other equilibria in

⁸The relationship follows from the assumptions of uniform prior and uniform noise because for $x_i \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$, $\mathbf{P}(x_i \leq x^* | \theta^*) = \frac{x^* - \theta^* + \varepsilon}{2\varepsilon} = 1 - \frac{\theta^* - x^* + \varepsilon}{2\varepsilon} = 1 - \mathbf{P}(\theta \leq \theta^* | x^*)$.

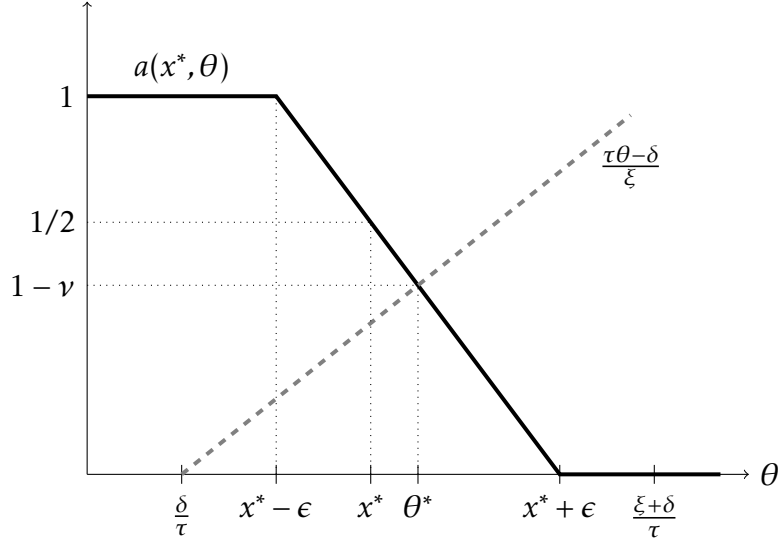


Figure 2: Determination of bank's failure point θ^* and the depositors' threshold x^* (without loss of generality, the figure assumes $\nu > 1/2$).

non-monotone strategies, i.e., all other strategy profiles can be ruled out by elimination of strictly dominated strategies. For completeness, the proof is contained in the Appendix.

Proposition 2 (Morris & Shin). *The unique equilibrium of the incomplete information game is such that each player i chooses action 1 if and only if $x_i < x^*$ and $R(a(x^*, \theta), \theta) < 0$ if and only if $\theta < \theta^*$. The equilibrium thresholds (θ^*, x^*) are given by*

$$\theta^* = \underline{\theta} \left(1 + \frac{\xi(1-\nu)}{\delta} \right) \in (\underline{\theta}, \bar{\theta}) \quad \text{and} \quad x^* = \theta^* - 2\epsilon \left(\nu - \frac{1}{2} \right). \quad (3)$$

The solution is illustrated graphically in Fig. 2. The figure plots condition (CM), split up between the aggregate action $a(x^*, \theta)$ (black solid line) and the remainder $\frac{\tau\theta - \delta}{\xi}$ (gray dashed line), against the fundamental θ . For $\theta < x^* - \epsilon$, all players obtain a signal below x^* and choose action 1. Conversely, for $\theta > x^* + \epsilon$, nobody obtains a signal below x^* and all choose action 0. For values of θ between $x^* - \epsilon$ and $x^* + \epsilon$, the aggregate action linearly decreases in θ . The critical point θ^* is at the intersection of the two curves. From (IC) follows that the aggregate action at which the function $R(\cdot)$ changes sign, i.e., $a(x^*, \theta^*)$, is just equal to $1 - \nu$.

Limit of Vanishing Private Noise. It follows from [Proposition 2](#) that the threshold x^* and the failure point θ^* differ only by a constant which depends on the noise parameter ε . If ε becomes smaller, then x^* converges to θ^* . The variance in the players' posterior distribution shrinks and in the limit, where $\varepsilon \rightarrow 0$, all players almost perfectly observe the realized state. Players' behavior becomes perfectly synchronized: if $\theta < \theta^*$, all players observe $x_i \approx \theta < \theta^* = x^*$ and choose $a_i = 1$, inducing regime change; conversely, if $x_i \approx \theta > \theta^* = x^*$, all choose $a_i = 0$ and the status quo is maintained. As a consequence, the aggregate action converges to a step function with a discontinuity at θ^* .

Vanishing uncertainty about the true state, however, does not restore common knowledge about neither state, nor payoffs. [Carlsson and van Damme \(1993\)](#) point to the crucial difference between 'knowledge' and 'common knowledge' of the state. In the complete information game, players have common knowledge about θ , implying that they not only know the true state, but also share the same beliefs, know that everyone knows this and so on ad infinitum. In the incomplete information game, common knowledge is absent even if fundamental uncertainty (i.e., the signal noise) vanishes and the state becomes almost perfectly known to each player. Heuristically, this point can be illustrated with just two players. Conditional on their signal, each player believes that $\theta \in [x_i - \varepsilon, x_i + \varepsilon]$. Given x_1 , player 1 reasons that player 2's signal $x_2 \in [x_1 - 2\varepsilon, x_1 + 2\varepsilon]$. Hence, player 1 believes that player 2 believes that $\theta \in [x_1 - 3\varepsilon, x_1 + 3\varepsilon]$. Furthermore, player 1 believes that player 2 believes that player 1 believes that $\theta \in [x_1 - 5\varepsilon, x_1 + 5\varepsilon]$. If this process is repeated to obtain beliefs of ever higher order, then the length of the intervals on which players base their inferences about what the other player believes that she believes etc. stretch further out, even if ε becomes small. As a consequence, even though uncertainty about the state may become vanishingly small, strategic uncertainty, i.e., uncertainty about what the other players know and how they play in the game, does not vanish.⁹

⁹[Heinemann et al. \(2009\)](#) use global games to model strategic uncertainty and to devise a method

Laplacian Belief. What are players’ beliefs about their opponents’ behavior in the global game? The answer to this question is surprisingly simple. In particular, the player who observes the threshold signal x^* believes that the aggregate action is uniformly distributed over $[0, 1]$. Thus, she treats strategic uncertainty as if she was completely unsure about the behavior of other players and applied Laplace’s ‘principle of indifference’ that attaches equal weight to all mutually exclusive outcomes. [Morris and Shin \(2003\)](#) refer to this belief as a ‘Laplacian belief’ that “*represents an agnostic or diffuse view on the actions of other players in the game*” (p. 15).

The Laplacian belief is a consequence of players’ rank-order beliefs. The rank-order belief is a player’s belief about the rank of her private signal in the population. Under the uniform prior, it turns out that a player’s rank-order belief is uninformative and provides no information about her rank in the overall distribution. To see this, consider the player with signal x_i . Her rank-order belief is the probability she attaches to the share of others who observe a smaller signal. Given θ , the share of other players with a signal smaller than x_i is equal to $a(x_i, \theta)$. Treating θ as a random variable, the rank-order belief of player i is the probability, conditional on x_i , that $a(x_i, \theta)$ is less than some value ℓ ,

$$\mathbf{P}(a(x_i, \theta) \leq \ell | x_i) = \mathbf{P}(\theta \geq x_i + \varepsilon - 2\varepsilon\ell | x_i) = 1 - \frac{(x_i + \varepsilon - 2\varepsilon\ell) - (x_i - \varepsilon)}{2\varepsilon} = \ell.$$

In words, the rank-order belief of the player with signal x_i is independent of her signal and uniform over $[0, 1]$, implying that the player has no information about her rank in the distribution. In this sense, player x_i views herself as a typical player from the population of players ([Edmond, 2018](#)).¹⁰

In the global game, the rank-order beliefs immediately translate into beliefs about

for measuring strategic uncertainty experimentally.

¹⁰Notice that the uniform prior assumption is a sufficient (but not a necessary) condition for these properties to hold. Appendix B shows that a uniform rank-order belief also obtains in the Gaussian case when the precision of private information becomes sufficiently precise relative to the precision of the prior.

the aggregate action. The threshold player in the global game is the player who is indifferent between her two actions. The threshold signal x^* can then be pinned down as the signal that renders a player with a uniform belief about the aggregate action indifferent.

An important implication of this property is that the solution to a standard global game can be derived by applying a ‘Laplacian belief heuristic’:¹¹ (i) postulate that the aggregate action $s \sim U[0, 1]$; (ii) calculate the probability of regime change from (CM) as $1 - \frac{\tau\theta^* - \delta}{\xi}$; (iii) apply this probability to calculate the indifference condition for a player and solve the respective equation for θ^* .¹²

2.4 Applications to Bank-Runs

Bank-run models are an important area of application for global games.¹³ The global game approach provides a tractable way of formalizing the fragility of financial institutions prone to runs and coordination failures. Furthermore, it allows to reconcile two opposing popular views on the origins of runs and bank failures (Tirole, 2002; Chui and Gai, 2005): (i) the ‘panic view’, whereby runs are the outcome of sunspots or self-fulfilling beliefs that cause banks to fail due to the liquidity or maturity mis-

¹¹Frankel (2024) shows that the Laplacian heuristic implied by global games is but one of a larger class of equilibrium selection criteria for so-called ‘participation games’ (i.e., games where a critical mass of players must participate to make participation worthwhile). These criteria share the feature that players best-reply to a particular distribution over the proportion of other players who participate. These ‘fictional beliefs’ (for global games, the uniform distribution over $(0, 1)$) are common for all players and are independent of the payoff structure of the game.

¹²The ‘Laplacian belief’ of the threshold player holds in standard global games (Edmond, 2018). However, a number of important applications, e.g., the large player game by Corsetti et al. (2004), the game with heterogeneous groups by Sakovics and Steiner (2012), or the signalling game by Edmond (2013) are examples of non-standard global games where equilibrium beliefs differ from the Laplacian belief.

¹³The first applications to bank-runs are Morris and Shin (2000), Rochet and Vives (2004) and Goldstein and Pauzner (2005). Morris and Shin (2000) and Goldstein and Pauzner (2005) select a unique equilibrium in variants of the model by Diamond and Dybvig (1983). In their models, depositors’ payoffs are tightly related to the bank’s deposit liabilities and directly depend on the fundamental and the aggregate action. Rochet and Vives (2004), in contrast, use a payoff structure akin to the regime change game in Table 1 where payoffs are independent of the bank’s deposit liabilities. They motivate this assumption by interpreting players as fund managers who only administer deposits and are paid an exogenous compensation for making the ‘right’ decision, which is independent of the value of deposits under management.

matches on their balance sheet (Diamond and Dybvig, 1983); (ii) the ‘fundamental view’, whereby runs are a consequence of solvency concerns and (information about) bad economic fundamentals (Chari and Jagannathan, 1988; Jacklin and Bhattacharya, 1988).

The regime change game as a bank-run model. Following Rochet and Vives (2004) or Morris and Shin (2016), we can interpret the regime change model from Section 2.1 as a bank-run model. Players are depositors who decide on rolling over ($a_i = 0$) or withdrawing ($a_i = 1$) short-term or demandable deposits. Under the status quo, the bank survives, whereas the bank fails if the status quo is abandoned. The function $R(a, \theta)$ measures the bank’s equity value where θ is the return on the bank’s long-term assets. To service early withdrawals, the bank must sell assets prematurely at fire-sale prices. As withdrawal pressure, a , increases, so too do the fire-sale losses, which deplete the bank’s equity and can drive it into default. The assumption that $R(a, \theta)$ increases in θ implies that fire-sale losses are less severe for a bank with more profitable assets.

Liquidity and solvency. An important question in practice is whether the failure of a bank occurs due to ‘insolvency’ (which requires an orderly resolution procedure) or due to ‘illiquidity’ (which warrants policy support such as liquidity injections etc.). The tripartite classification of θ in Fig. 1 allows us to sharply distinguish insolvency from illiquidity. The bank is ‘insolvent’ if $\theta < \underline{\theta}$. In this case, asset returns are so low that failure is inevitable, irrespective of the size of deposit withdrawals. For $\theta > \bar{\theta}$, the bank is ‘liquid and solvent’. Its assets command enough liquidity to withstand a full-blown deposit withdrawal. For $\theta \in [\underline{\theta}, \bar{\theta}]$, the bank is ‘solvent but illiquid’. Even though it could meet all its liabilities absent a run, its returns are not large enough to withstand fire-sale losses when a run occurs. Thus, for $\theta \in [\underline{\theta}, \theta^*)$, the bank ‘fails due to illiquidity’.

The global games perspective shows that failure due to illiquidity, i.e., for $\theta \in [\underline{\theta}, \theta^*)$, is inextricably linked to the possibility of the bank's insolvency. To understand how this link arises, note that depositors' behavior is uniquely pinned down when they observe signals $x_i < \underline{\theta} - \varepsilon$ that ensure them that the bank is insolvent. For higher signals, depositors attach a strictly positive probability to the bank's solvency. Yet, they face uncertainty about what others know and take into account that other depositors may observe lower signals and believe that the bank is insolvent for sure. As a consequence, a depositor may withdraw even if she knows that the bank is solvent, just because she expects others to hold the opposite belief and act accordingly. More specifically, consider a depositor who observes $x_i \in (\underline{\theta} - \varepsilon, \underline{\theta})$ and holds the most optimistic belief that others withdraw only if they know for sure that the bank is insolvent. Given her signal, the depositor infers that some other depositors may observe signals below $\underline{\theta} - \varepsilon$ and she expects at least these depositors to withdraw. From her posterior she infers that θ is close to $\underline{\theta}$ and because of state monotonicity she knows that at this level of θ a few withdrawals suffice to bring down the bank. Thus, despite her most optimistic belief about others' behavior, she herself prefers to withdraw at a signal close to, but above $\underline{\theta} - \varepsilon$. Moreover, she knows that other depositors will act in the same way as she does. Because of strategic complementarities, no depositor will withdraw only for signals below $\underline{\theta} - \varepsilon$. Hence, our depositor must revise her most optimistic belief. This process is repeated so that the depositors' critical signals and the bank's failure point are successively revised upwards beyond $\underline{\theta}$. The very fact that the bank becomes insolvent at sufficiently low values of θ (where withdrawing is strictly dominant) 'infects' the behavior at larger values of θ where the bank is solvent and eventually can cause a solvent institution to fail.¹⁴

Implications for prudential regulation and policy. An important consequence of the tight connection between liquidity and solvency is that changes to a bank's sol-

¹⁴Formally, this infection process corresponds to the iterated deletion of dominated strategies, see the proof of [Proposition 2](#).

vency risk also always affect its liquidity risk. This can be illustrated with [Eq. \(3\)](#) which shows that any change to the lower dominance bound $\underline{\theta}$ (which determines solvency) exerts an effect on the failure point θ^* that is amplified by a factor $(1 + \xi(1 - \nu)/\delta) > 1$, which increases in the degree of strategic complementarities, ξ . Similarly, [Rochet and Vives \(2004\)](#) and [Vives \(2014\)](#) show that banks' capital ratios not only mitigate solvency but also liquidity risk. [Koenig \(2015\)](#) shows that the connection between solvency and liquidity implies that liquidity ratios (measured as cash over deposits) lead to counteracting effects. Higher liquidity ratios reduce pure liquidity risk, measured by $\theta^* - \underline{\theta}$, because they mitigate fire-sale losses. But they increase solvency risk, measured as $\underline{\theta}$, because holding cash requires foregoing more profitable investments, which further affects liquidity risk so that the total effect is ambiguous.

Even though the comparative statics from the regime change model provide useful indications for the joint effects of capital and liquidity regulation, the basic model presented above is a partial equilibrium model that takes the players' payoffs and the bank's balance sheet as given. It is therefore not suited to study questions pertaining to optimal regulations and their welfare implications. A number of recent studies embed bank-run global games into richer models that endogenize the bank's balance sheet and the face value of its debt and deposits to express optimal policies solely in terms of structural parameters.

Broadly speaking, the bank's balance sheet variables and depositors' payoffs in these models are determined by trading off changes in (bank) profits or social welfare against changes in the likelihood of a run that stem from the dependency of the global game threshold θ^* on the balance sheet variables or the face value of debt. [Rochet and Vives \(2004\)](#) and [Eisenbach \(2017\)](#) exploit this relationship to understand to what extent the view that short-term debt can act as a device to discipline the bank manager holds up in the presence of inefficient runs. [Ahnert et al. \(2018\)](#) derive optimal asset encumbrance regulation for secured debt claims by studying how banks'

secured debt issuance impacts liquidity risk when the assets backing secured debt are not available to service early withdrawals. [Carletti et al. \(2020\)](#), [Ikeda \(2024\)](#) and [Kashyap et al. \(2024\)](#) endogenize the liability and the asset side of a bank to study joint liquidity and capital regulations. [Carletti et al. \(2020\)](#) and [Ikeda \(2024\)](#) derive conditions under which capital regulation alone can suffice to induce the bank to implement the efficient level of liquidity. In the model by [Kashyap et al. \(2024\)](#) capital and liquidity regulations can, in general, implement the socially optimal risk level, but at the expense of depressing credit creation.

3 The Effects of Size and Heterogeneity

The previous section focused on a regime change model with homogeneous players (aside from differences in their private information). In particular, (i) each player's action has the same, negligible impact on the status quo; (ii) each player faces the same payoffs, so that, for a particular signal, players' incentives to choose a certain action are the same. In this section, we extend the analysis of our baseline more to include large players and players with heterogeneous payoffs.

3.1 Size

Regime change with a large player. [Corsetti et al. \(2004\)](#) relax the assumption that all players have the same negligible impact on the status quo. Instead, they consider a model with a large player, whose action has a disproportionately large (and sometimes pivotal) effect on the status quo, operating alongside a continuum of small players.

A key result of their analysis is that the large player, in general, improves coordination and increases the likelihood that the 'efficient outcome' obtains in equilibrium.¹⁵

¹⁵In the multiple equilibrium region of their standard currency attack game, the equilibrium where the currency peg breaks down is the efficient (Pareto-dominant) outcome. Thus, in this game, the large player increases the likelihood that the peg breaks down. Conversely, in a bank-run model, the failure

In particular, [Corsetti et al. \(2004\)](#) disentangle a ‘size effect’ and an ‘information effect’ of the large player. The former refers to the effect of the size of the large player on the incidence of an attack.¹⁶ The latter relates to the effect of the precision of the private information of the large player compared to the information precision of the smaller players.

To illustrate these two effects, consider the benchmark regime change model in [Section 2.3](#) and suppose that a large player with mass $\gamma \in (0,1)$ operates alongside a continuum of small players of mass $1 - \gamma$. The large player receives a signal $y = \theta + \tilde{\omega}$, where $\tilde{\omega} \sim U[-\omega, \omega]$, and the small players receive signals as in [Section 2.3](#). The payoff parameters are the same for the small and the large player. Without loss of generality, we focus on threshold strategies around y_L^* (large player) and x_L^* (small players).

General closed-form solutions for the large player model are difficult to obtain, even under the simplifying assumption of uniform noise. In the limit, where ε and ω vanish, the solution depends on the ratio of the noise of the large and the small traders’ signals. Thus, for $\varepsilon \rightarrow 0$ and $\omega \rightarrow 0$, an assumption about the convergence of the ratio $\frac{\varepsilon}{\omega}$ is required. Closed-form solutions can be obtained for two extreme cases where either the large player is better informed ($\frac{\varepsilon}{\omega} \rightarrow \infty$) or the small players are better informed ($\frac{\varepsilon}{\omega} \rightarrow 0$).

Proposition 3 (Corsetti, Dasgupta, Morris & Shin). *For $\varepsilon \rightarrow 0$, $\omega \rightarrow 0$, the unique equilibrium with a large player of size γ is such that the large player chooses action 1 if and only if $y < y_L^*$, the small players choose action 1 if and only if $x_i < x_L^*$, and the status quo is*

of the bank is usually the suboptimal (Pareto-dominated) outcome. For instance, [Chen et al. \(2010\)](#) show that a large institutional investor in a fund can reduce the risk of redemption runs. However, [Liu and Mello \(2017\)](#) argue that a large player can also be destabilizing. In their model, the large player, different from the small players, faces a risk management constraint. If this constraint is binding, the presence of a large player hastens a run. [Corsetti et al. \(2006\)](#) interpret the large player as the lender-of-last-resort (LOLR). They show that liquidity injections by the LOLR can reduce the risk of a run, but also may incentivize moral hazard by lowering incentives to reduce asset risk.

¹⁶[Corsetti et al. \(2004\)](#) show that in general, the size effect can be positive or negative. They distinguish between a global size effect, i.e., the presence of a large player; and a local size effect, i.e., the effect of marginally changing the size of the large player.

abandoned if and only if $\theta < \theta_L^*$.

1. If the large player is relatively better informed, $\frac{\varepsilon}{\omega} \rightarrow \infty$, then

$$x_L^*, y_L^* \rightarrow \theta_L^* = \underline{\theta} \left(1 + \frac{\xi(1-\gamma)(1-\nu)}{\delta} \right). \quad (4)$$

2. If the small players are relatively better informed, $\frac{\varepsilon}{\omega} \rightarrow 0$, then

$$x_L^*, y_L^* \rightarrow \theta_L^* = \begin{cases} \underline{\theta} \left(1 + \frac{\xi(1-\gamma)(1-\nu)}{\delta} \right) & \text{if } \gamma > 1 - \nu \\ \underline{\theta} \left(1 + \frac{\xi(1-\nu)}{\delta} \right) & \text{if } \gamma \leq 1 - \nu \end{cases} \quad (5)$$

It can be immediately read off from Eqs. (4) and (5) that the thresholds collapse to the threshold in Proposition 2 if the large player is absent, i.e., if $\gamma = 0$.

Size effect of large player. The size effect of the larger player is (weakly) negative for the two cases shown in Proposition 3. Whenever the large player is better informed, Eq. (4) implies that an increase in the size of the large player (γ) strictly lowers the failure point towards the lower dominance bound $\underline{\theta}$.¹⁷ If the small players are better informed, then the size effect is present and the large player exerts an effect only if $\gamma > 1 - \nu$, i.e., only if the large player has sufficient weight.

Information effect of large player. The information effect can be seen by comparing the thresholds in Eqs. (4) and (5). If the large player is relatively better informed, then the failure threshold is strictly smaller. The reason is that if the large player has a relatively precise signal, the small players treat the information of the large player as if it perfectly coincided with the true state and they condition the calculation of their expected payoff only on their own signal.

¹⁷Notice that in a single player decision problem with only one player ($\gamma = 1$), the large player would attack if and only if $\gamma < \frac{\delta}{\tau} - 2\omega(\nu - \frac{1}{2})$, which converges to $\underline{\theta}$ for $\omega \rightarrow 0$.

3.2 Heterogeneity

Regime change with heterogeneous groups. Whereas [Corsetti et al. \(2004\)](#) focus on differences in size, [Sakovics and Steiner \(2012\)](#) consider groups of small players where payoff parameters and the impact on the aggregate action differ across groups. The question addressed in their model is which player (i.e., groups) should be targeted by interventions of a benevolent planner to improve coordination towards the efficient outcome. They show that optimal (constrained) interventions target players with a strong influence on others' behavior, but who are themselves rather insensitive to others' actions.

The fundamental property that gives rise to their result is the so-called 'aggregate belief constraint', the extension of the Laplacian belief to a situation with heterogeneous groups of players. While the Laplacian belief states that the threshold player's belief about the share of others who attack is uniform on $[0, 1]$, the aggregate belief constraint states that the (weighted) average belief over all groups' threshold players is uniform on $[0, 1]$. The belief constraint allows to easily compute the critical failure point for the limit case of vanishing noise, similarly to the Laplacian belief heuristic discussed in [Section 2](#).

Besides offering a convenient way of calculating the equilibrium threshold, the belief constraint also provides a deeper insight into the distribution of optimism and pessimism in the population. To see this, suppose there are two groups of players who, because of differences in their payoff functions, switch from attack to refrain at different threshold signals, x_1^* and x_2^* , with $x_1^* < x_2^*$. The player who observes the threshold signal x_1^* is pessimistic about the behavior of players from group 2 because she expects them to attack. The reason is that with vanishing noise, she expects signals to be clustered around x_1^* and therefore below x_2^* . For the same reason, however, the threshold player in group 2 is more optimistic about the behavior of group 1 players.

Sakovics' and Steiner's aggregate belief constraint states that the pessimism of group 1 is balanced by the optimism in group 2: “*optimism about the aggregate action [can be treated] as a virtual resource, which is available in a fixed amount and distributed among the groups*” (Sakovics and Steiner, 2012, p. 3448).

This result bears an important lesson for understanding the design of optimal policies to mitigate coordination problems such as bank-runs or market dry-ups. In light of the aggregate belief constraint, policy interventions can be viewed as a re-distribution of strategic optimism across different groups of players. The optimal policy intervention in Sakovics and Steiner (2012) minimizes the risk of a coordination failure by re-shuffling strategic uncertainty from groups that strongly react to it to groups that do not.

We provide a brief summary of the model by Sakovics and Steiner (2012). Assume that the continuum of players is of total mass M and there are G different groups of players, indexed by $g \in \{1, \dots, G\}$. Groups differ by the payoff parameter of their members, $v_g \in (0, 1)$. The group size is denoted by m_g , i.e., $\sum_g m_g = M$. The aggregate action is

$$a = \int_{i \in M} w_i a_i,$$

where w_i is the weight of an agent i on the aggregate action. The weights are identical for players from the same group, i.e., $w_i = w_g = w_j$ for i, j being members of group g . For convenience, $\sum_g m_g w_g \equiv 1$. The information structure and the critical mass condition remain the same as in the benchmark model in Section 2.

Proposition 4 (Sakovics & Steiner). *The unique equilibrium of the game with G groups is such that each player i from group g attacks if and only if $x_{g,i} < x_g^*$ and $R(a, \theta) < 0$ if and only if $\theta < \theta^*$. In the limit, for $\varepsilon \rightarrow 0$, $x_g^* \rightarrow \theta_S^*$ for all g , where*

$$\theta_S^* = \underline{\theta} \left(1 + \frac{\xi \left(1 - \sum_{g=1}^G m_g w_g v_g \right)}{\delta} \right). \quad (6)$$

It can be immediately seen from Eq. (6) that θ_S^* equals the threshold θ^* in Proposition 2 if $\frac{C_g}{B_g} = \nu_g = \nu = \frac{C}{B}$ for all g .¹⁸

Optimal intervention scheme to reduce coordination risk. Consider a benevolent planner who wants to implement a certain threshold $\theta^+ \in (\underline{\theta}, \theta_S^*)$. If the planner could costlessly manipulate individual cost-benefit ratios, e.g., by changing the cost parameter C_g via a tax, then there exist infinitely many equally optimal intervention schemes that all implement θ^+ .

However, as Sakovics and Steiner (2012) show, if the planner faces a constraint, e.g., keeping a minimum reservation utility for players or a budget constraint etc., then the optimal intervention scheme in general targets those players (i.e., groups) who have the largest w_g/B_g -ratios. These are groups whose players exert a strong influence on the aggregate action (w_g large) but who are rather insensitive to strategic uncertainty and generally less inclined to attack (B_g small).

4 Interlinkages and the amplification of runs

The canonical regime change model can be used to describe the individual fragility of banks or other institutions. In practice, however, different institutions may be interlinked through markets and other arrangements. Examples of such links include secondary markets where banks alongside other market participants buy and sell assets, exchange rate regimes where a government pegs its currency while banks hold foreign currency denominated debt, or official sector guarantees to banks that, when triggered, can jeopardize the government's own finances. In each of these examples, an attack (run) against one institution can increase the regime change risk for other

¹⁸Szkup (2020) provides general comparative statics results for global games in the Sakovics-Steiner framework. He shows that the effect of a parameter change can be decomposed into a direct effect and an amplifier effect that measures the impact of the adjustments in the beliefs of the threshold players of the different groups.

institutions, which can feed back and exacerbate the risk of regime change for all institutions.

To capture these possibilities, we extend the canonical regime change model for one institution to include two (identical) institutions, $j \in \{1, 2\}$ each with its own distinct unit continuum of players. The strength of the status quo for both institutions depends on the same common fundamental, θ . Generalizing the payoffs from Table 1, the payoff difference, π_i^j for player i in institution j is given in the following Table.

	$R^j(a^j, a^{-j}\theta) < 0$	$R^j(a^j, a^{-j}, \theta) > 0$
$a_i = 1$	$B - C$	$B - C$
$a_i = 0$	0	B

Table 2: Payoff matrix for the game with interlinkages

$R^j(a^j, a^{-j}, \theta)$ represents the strength of the status quo at institution j , which depends on the mass of players, a^j , who attack institution j , and on the mass of players, a^{-j} , who attack the other institution, $-j$. We assume

$$R^j(a^j, a^{-j}, \theta) = \tau\theta - \xi a^j - \zeta a^{-j} - \delta, \quad (7)$$

where the parameters satisfy $\tau > 0$, $\xi > 0$, $\zeta > 0$ and $\delta > 0$. The term ζ captures the impact that the attack on institution $-j$ has on institution j . For instance, in the secondary market example, ζ may capture the negative price impact that the sale of assets by one institution has on the other institution. Even though the linear separable mapping in Eq. (7) is purposefully stylized, our modeling is broadly consistent with the models studied in the literature.

To solve the model, we utilize the lines of reasoning developed by Goldstein (2005). In particular, the first step is to characterize how the threshold strategy used by players of institution 1 depends on that used by players of institution 2 and vice versa. These strategies are derived by solving the system of indifference and critical mass conditions for each of the two games separately to obtain the threshold signal for

players in institution j , x_j^* , as a function of the threshold used by players of institution $-j$, x_{-j} . For the regime change game on institution j , the indifference and critical mass conditions are

$$\mathbf{P}\left(\theta \leq \theta_j^* \mid x_j^*\right) - \nu = 0, \quad (IC_j)$$

$$\tau \theta_j^* - \xi a(x_j^*, \theta_j^*) - \zeta a(x_{-j}, \theta_j^*) - \delta = 0. \quad (CM_j)$$

The critical mass condition depends on the mass of players who attack the other institution, $a(x_{-j}, \theta_j^*)$, where x_{-j} is the threshold used by players of institution $-j$. Importantly, since the payoff does not depend on the outcome for the other institution, i.e., whether the regime changed or not, the critical mass condition does not depend on the other institution's failure threshold.

Figure 4 depicts the two equilibrium threshold strategy schedules. Because $\zeta > 0$, there are strategic complementarities across the two institutions, implying that the two schedules are strictly increasing. For intuition, consider the behavior of agents under *extreme beliefs*. Let $\tilde{x}_1$ be the threshold used by players of institution 1 who believe that all institution 2 players attack, i.e., the threshold derived under the assumption that $a_2 \equiv 1$. Similarly, define $\hat{x}_1$ as the threshold when they instead believe that $a_2 = 0$. Because $\zeta > 0$, $\hat{x}_1 < \tilde{x}_1$, i.e., the players of institution 1 are more likely to attack their institution if they expect all players of institution 2 attack their institution as well. Analogously, we conduct a similar exercise for the players of institution 2 and derive $\tilde{x}_2$ and $\hat{x}_2 < \tilde{x}_2$. Figure 4 overlays these extreme belief thresholds on agents' threshold strategies.

Now suppose that players of institution 1 have very optimistic beliefs about the stability of institution 2. And so they believe that irrespective of the signals received by institution 2 players, they always refrain from attacking. In this case, players of institution 1 use the threshold $\hat{x}_1$. Given $\hat{x}_1$, players of institution 2 use $x_2^*(\hat{x}_1)$ as their own strategy. And, in turn, anticipating this, players of institution 1 update their

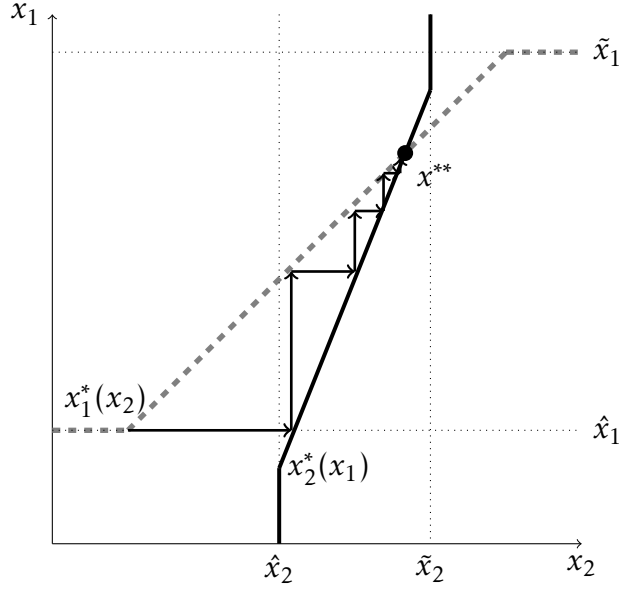


Figure 3: Threshold strategies.

threshold to $x_1^*(x_2^*(\hat{x}_1))$, and so forth. This iterative process continues until a fixed point is reached where $x_1^*(x_2^*) = x^{**} = x_2^*(x_1^*)$. Proposition 5 characterizes this equilibrium.

Proposition 5. *Suppose that institutions are identical, $\hat{x}_1 = \hat{x}_2 = \hat{x}$ and $\tilde{x}_1 = \tilde{x}_2 = \tilde{x}$. Players attack if and only if $x_i < x^{**}$ and the status quo breaks down for both institutions if and only if $\theta < \theta^{**}$. In the limit, for $\epsilon \rightarrow 0$, the equilibrium signal and the failure threshold converge, $x^{**} \rightarrow \theta^{**}$, where*

$$\theta^{**} = \underline{\theta} \left(1 + \frac{(\xi + \zeta)(1 - \nu)}{\delta} \right). \quad (8)$$

With identical institutions, the extreme beliefs held by players across the two institutions are identical. The ordering of the extreme beliefs, $\hat{x} < \tilde{x}$, therefore implies that regime change across the two institutions is perfectly correlated. Moreover, the joint equilibrium, x^{**} , lies in the interval $(\hat{x}, \tilde{x})$ as depicted in Figure 4.

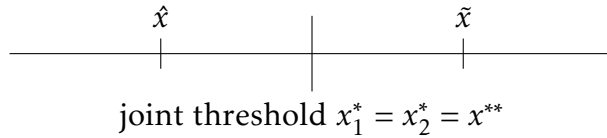


Figure 4: Ordering of critical signals and correlation of attacks.

Interlinkages across institutions engenders greater fragility for all institutions. This

level effect is a consequence of cross-institution strategic complementarities that has previously been identified in a number of studies.

[Goldstein \(2005\)](#) shows that in the presence of a currency peg, speculative attacks on the currency can amplify runs when banks are funded by foreign denominated debt. And, greater runs on banks places greater pressure on the central bank's reserves to defend the currency peg, which heightens the incentives of speculators to attack. [Koenig and Pothier \(2022\)](#) argue that across-institution strategic complementarities can arise from information acquisition incentives by non-bank financial institutions and the run incentives of their shareholders. In equilibrium, endogenous adverse selection and redemptions can amplify and lead to simultaneous market and funding liquidity dry-ups.

[Eisenbach \(2017\)](#), [Goldstein et al. \(2022\)](#) and [Liu \(2023\)](#) study bank runs where banks are connected via fire-sales in common asset markets. While the papers differ in their approaches to modeling fire-sale prices, they all share the feature that the price is decreasing in the amount of assets being sold. This is, in effect, the source of the across-bank strategic complementarity that increases the proclivity for runs across banks.

[Anand et al. \(2012\)](#) and [Liu \(2016\)](#) consider banks that are connected via interbank markets. In these models, shocks to banks can cause their lenders to lose confidence and withdraw. Due to the linkages, this loss in confidence can spread across the entire interbank network leading to a market freeze.

[Koenig et al. \(2014\)](#) and [Leonello \(2018\)](#) consider how government guarantees to banks create a two-way feedback between bank-runs and sovereign debt crises. When fewer sovereign creditors roll over their sovereign debt claims, the guarantee becomes less credible, which, in turn, incentivizes bank depositors to run. Depending on the design of the guarantee scheme, it is possible that, as more depositors withdraw, the government must transfer more resources to the banking sector, which impinges

on the government's solvency and increases sovereign creditors' incentives to run as well.¹⁹

Heterogeneous institutions. The analysis thus far focused on homogeneous institutions. In what follows, we extend the model to heterogeneous institutions. We model differences in institutions via differences in the reduced-form parameters δ and τ . In particular, we assume $\delta_2 > \delta_1$ and $\zeta_2 > \zeta_1$. This implies that (i) institution 1 is stronger and less susceptible to a regime change than institution 2 and (ii) institution 1 is less impacted by the attack on institution 2 than institution 2 is impacted by the attack on institution 1.

Proposition 6. *Suppose $\delta_2 > \delta_1$, and $\zeta_2 > \zeta_1$. If*

$$\zeta_1 > \delta_2 - \delta_1,$$

then the extreme beliefs satisfy the ordering

$$\hat{x}_1 < \hat{x}_2 < \tilde{x}_1 < \tilde{x}_2,$$

*and regime change across the two institutions is perfectly correlated. Defining $\alpha \equiv \frac{\zeta_2}{\zeta_1 + \zeta_2}$, in the limit of vanishing private noise, $\epsilon \rightarrow 0$, the joint equilibrium threshold converges to the failure threshold, $x^{**} \rightarrow \theta^{**}$, such that*

$$\theta^{**} = \alpha \theta_1^{**} + (1 - \alpha) \theta_2^{**} + \frac{2(1 - \nu) \zeta_1 \zeta_2}{\tau(\zeta_1 + \zeta_2)}, \quad (9)$$

where

$$\theta_j^{**} = \frac{\delta_j + (1 - \nu) \xi}{\tau},$$

¹⁹Koenig et al. (2014) argue that the larger is the mass of depositors who withdraw, the smaller is the pool of depositors who receive the guarantee, which then improves sovereign creditors' incentives to roll over government debt. Thus, the actions of sovereign creditors are strategic complements for bank depositors, but the actions of bank depositors are strategic substitutes for sovereign creditors.

for all $j \in \{1, 2\}$. It follows that $\frac{\partial \theta^{**}}{\partial \delta_1} > \frac{\partial \theta^{**}}{\partial \delta_2}$.

The ordering of the extreme beliefs implies that regime changes across the two institutions are perfectly correlated as in the case of homogeneous institutions.²⁰ In particular, it ensures that the intersection between the best responses correspondences, $x_1^*(x_2)$ and $x_2^*(x_1)$ occurs in the region where both are increasing as depicted in Figure 4. As a consequence, we can express the equilibrium critical threshold as a weighted sum of the critical thresholds for the two institutions.

The linkage between the two institutions imparts two effects. First, the status quo regime for both institutions is rendered more fragile, which is reflected by the last term in Eq. (9). And second, there are policy spillovers whereby a policy change to institution 1 also influences institution 2. In particular, a marginal increase in δ_1 induces a larger negative impact on the joint threshold than a corresponding increase in δ_2 . In other words, in the presence of interlinkages, strengthening the ‘stronger’ institution (reducing δ_1) imparts greater stability than strengthening the ‘weaker’ one (reducing δ_2).

Proposition 6 is consistent with recent results on the role of heterogeneity and interlinkages in bank- and market-runs. Choi (2014) shows that, in the presence of interlinkages, a run on a bank with strong fundamentals can trigger a run on a weaker bank as well, thereby triggering a system-wide crisis. But the converse need not be true. An important corollary of this result is that policy changes that strengthen the stronger bank improve system-wide stability by more than strengthening the weaker bank. Goldstein et al. (2022) obtain a related result on the role of heterogeneity in the context of bank runs, emphasizing that the reinforcement of within- and cross-bank strategic complementarities, which is absent in Choi (2014), can make heterogeneity beneficial for all.

²⁰If the parameter assumption in Proposition 5 fails to hold, then the ordering of extreme beliefs changes. As a consequence, the amplification between the attacks on the two institutions is weaker, see Goldstein (2005) for a full characterization.

5 Dynamic Coordination Games

The previous discussion centered around static (or one-shot) games of regime change. As pointed out by [Angeletos and Lian \(2016\)](#), the static approach abstracts from the inherently dynamic nature of crises phenomena where players either have multiple attempts to attack the status quo or obtain their chance to attack at different points in time. Furthermore, the static model cannot properly take into account that players may learn from previous unsuccessful attacks about the strength of the status quo.

A detailed discussion of the growing literature that studies dynamic regime change models is beyond the scope of the present chapter. Here, we content ourselves with a few remarks on the different directions that this literature has evolved into.

[Morris et al. \(1999\)](#) is a precursor to dynamic global games. Even though the fundamental in their model evolves according to a stochastic process, the game boils essentially down to a succession of static games. [Dasgupta \(2007\)](#) introduces a two-period model where players can benefit from the option to delay their action. When delaying, players can learn by observing a noisy private signal about the size of the previous attack. Dasgupta shows that the option to delay reduces the incidence of a coordination failure.²¹ [Angeletos et al. \(2007\)](#) consider repeated attacks against the status quo whereby the fundamental remains unchanged over time. The dynamic learning process in their model is based on players not only observing new private signals every period but also learn about the strength of the status quo from previous unsuccessful attacks. This latter observation is public (i.e., common knowledge) and can undermine the subtle role of information in global games and bring multiple equilibria back.

One caveat of dynamic coordination games concerns the nature of strategic complementarities. In static global games, strategic complementarities are key to show

²¹Other models where learning is based on private signals are [Kovac and Steiner \(2013\)](#) who study the possibility of action reversibility in a dynamic setting and [Dasgupta et al. \(2012\)](#) who provide a characterization of players' behavior in terms of higher order beliefs.

the existence of a monotone equilibrium. [Echenique \(2004\)](#) argues that dynamic (extensive form) games with strategic complementarities are a rather restrictive class of games that renders analysis rather intractable. To overcome this restriction, [Mathévet and Steiner \(2013\)](#) develop a class of tractable dynamic games where they can characterize monotone equilibria without strict intertemporal complementarities.

Another strand of the literature on dynamic coordination games evolved separately, based on the ideas of [Frankel and Pauzner \(2000\)](#) and [Burdzy et al. \(2001\)](#).²² In these models the fundamental evolves according to a Brownian process and its realization at each point in time is common knowledge. However, players cannot constantly adjust their actions but only occasionally according to a Poisson clock. This timing friction prevents the synchronization of players' behavior and allows to select a unique equilibrium. The selection process is closely related to the 'infection' process that obtains in static global games of regime change. [Morris \(2014\)](#) offers a detailed discussion of the relationship between information frictions in static global games and the timing friction in dynamic games.²³

6 Conclusion

This chapter offers a cursory overview of the literature on global games of regime change. We intentionally stripped the discussion from detailed game-theoretic and technical considerations and focused on the simplest model and its variations to illustrate the mechanics, analytics and intuition behind the global game approach. Our choice of topics in this chapter, (i) canonical regime change model, (ii) size and heterogeneity, (iii) interlinkages, was guided by what we thought would prove to be a useful toolkit for practitioners in central banks and policy institutions.

²²[Guimaraes et al. \(2020\)](#) provide an introduction to the techniques and an overview over different applications.

²³Other papers and applications that build on the idea to use timing frictions to select a unique equilibrium in dynamic coordination games are [Guimarães \(2006\)](#), [Daniels \(2009\)](#), [He and Xiong \(2012\)](#), [Guimaraes and Machado \(2018\)](#), [Plantin and Shin \(2018\)](#) or [Guimaraes and Jardanovski \(2022\)](#).

The canonical regime change model offers a simple, albeit useful framework to understand the fragility of financial institutions. The extension to heterogeneous players gears the canonical model towards applications in financial stability or prudential regulation that address questions about the (de-)stabilizing role of large institutional investors, differences in the compensation and incentive structures of trading companies or the effect of liabilities with different repayment characteristics. The extension to models with interlinkages serves as a bridge from the canonical model towards richer models that address questions about systemic stability, contagion and macroprudential regulation.

In particular, these extensions highlight how strategic uncertainty among agents in the financial system can amplify the propagation of shocks through the financial and institutional links between intermediaries and markets. The global games approach thereby offers a specific view on the macroprudential tenet that *system-wide risk is greater than the sum of the risks faced by individual institutions*. Consequently, macroprudential policies should take into account the presence of cross-institution strategic complementarities engendered by such interlinkages. In other words, the presence of cross-institution strategic complementarities is an important *raison d'être* for macroprudential policies.

What is more, the extended model with heterogeneous agents and cross-institution linkages also offers some guidance for how to mitigate the effect of cross-institution strategic complementarities by highlighting the importance of certain key players and institutions. That is, to be effective, macroprudential policies should not be designed in a 'one-size-fits-all' approach, but rather target institutions and / or players that exert the largest impact on mitigating strategic uncertainty. Interestingly, these players are not necessarily those that identify as the weakest links in a financial system. For example, in a system with 'strong' and 'weak' institutions, bolstering the strong institutions to shore up their stability helps in mitigating the de-stabilizing effects stem-

ming from the cross-institution strategic complementarities by more than targeting the weak institutions.

Thus, even though the global games approach does not necessarily contradict how standard macroprudential policy tools (such as capital add-ons for large institutions, specific disclosure requirements for certain groups of institutions etc.) are employed, it provides a nuanced view on which players and institutions are key for effectively mitigating contagion effects and strategic uncertainty and should, therefore, become central to macroprudential analysis.

We hope that this chapter offers sufficient grounding for an interested reader to boldly continue their exploration of global games, to find new applications and insights. Perhaps the most important advice we can provide in this regard would be, in the immortal words of Douglas Adams' original hitchhiker's guide, "Don't Panic"!

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A Proofs

Proof of Proposition 2. We provide the standard argument that the unique monotone equilibrium (θ^*, x^*) is the only equilibrium of the regime change game, i.e., there are no other equilibria in non-monotone strategies. The argument proceeds by iteratively eliminating dominated strategies.

Define the function $\hat{\theta} : \hat{x} \rightarrow \Theta$ that provides the value of θ at which condition (CM) is satisfied with equality, given that players use a switching strategy around $\hat{x}$. Note that $\hat{\theta} \in [\underline{\theta}, \bar{\theta}]$ because $s \in [0, 1]$. $R(\hat{x}, \theta)$ is continuous, decreasing in $\hat{x}$ and increasing in θ . An application of the implicit function theorem shows that $\hat{\theta}$ increases in $\hat{x}$ due to strategic complementarities ($\xi > 0$).

Consider a typical player i with signal x_i who believes that other players use the monotone strategy around $\hat{x}$. Write i 's expected payoff difference as

$$\pi_i^e(x_i, \hat{x}) = \mathbf{P}(\theta \leq \hat{\theta}(\hat{x}) | x_i) - v,$$

and notice that π^e is continuous, decreases in x_i and increases in $\hat{x}$. Define the function $b : \hat{x} \rightarrow x_i$ as the solution to

$$\pi^e(x_i, \hat{x}) = 0,$$

i.e., $b(\hat{x})$ provides the signal x_i at which player i is indifferent between attacking and not attacking, given that other players use the strategy around $\hat{x}$. The monotone equilibrium in Proposition 2 is the unique fixed point of b , i.e., $x^* = b(x^*)$. By the implicit function theorem $b'(\hat{x}) \propto \hat{\theta}'(\hat{x})$ which is positive due to strategic complementarities.

Consider a critical signal $\underline{x}_0 = \underline{\theta} - \epsilon$. Because of the existence of dominance regions, a player with a signal $x_j < \underline{x}_0$ would always attack because she knows for sure that status quo regime will be abandoned. Thus, for signals below $\underline{x}_0$, attacking is the strictly dominant action. Suppose that players use the strategy around $\underline{x}_0$, i.e., they

always refrain from attacking, except if they are certain that the status quo regime will be abandoned. Given this aggregate strategy, player i becomes indifferent between attacking and not attacking at signal $\underline{x}_1 = b(\underline{x}_0) > \underline{x}_0$. Because of the monotonicity of b , for player i (and a fortiori for any other player), not attacking is dominated below $\underline{x}_1$. Continuing this process, we can define the increasing sequence

$$\{\underline{x}_n, n = 0, 1, \dots | \underline{x}_0 = \underline{\theta} - \varepsilon, \underline{x}_n = b(\underline{x}_{n-1}) \forall n = 1, 2, \dots\}.$$

Notice that the best-reply b is continuous and that the sequence is bounded above by the fixed point x^* . Every bounded monotonic sequence is convergent. Denote the limit of the sequence by $\underline{x}$. The limit is a fixed point of b , i.e., $\underline{x} = b(\underline{x})$. But since x^* is the unique fixed point of b , we must have $\underline{x} = x^*$. By the same logic, we can construct a decreasing sequence

$$\{\bar{x}_n, n = 0, 1, \dots | \bar{x}_0 = \underline{\theta} + \varepsilon, \bar{x}_n = b(\bar{x}_{n-1}) \forall n = 1, 2, \dots\}.$$

that also converges to x^* . The sequences are constructed in such a way as to eliminate strategies that are strictly dominated. The increasing sequence $\{\underline{x}_n\}$ successively eliminates rolling over for any signal below $\underline{x}_n$. The decreasing sequence successively eliminates withdrawing for any signal above $\bar{x}_n$. Since both sequences converge to the same limit, not attacking (attacking) is strictly dominated below (above) x^* . $\square$

Proof of Proposition 3. We focus on threshold strategies for the small and the large player. We show the existence and characterization of a unique threshold equilibrium. The proof that no other equilibrium in non-threshold strategies exists follows the same logic as in the proof of Proposition 2 and can be found in Corsetti et al. (2004). Suppose that the small players follow a symmetric threshold strategy around x^* and the large player uses a threshold strategy around y^* . There are two failure points, depending on whether or not the large player attacks. First, if the large player

refrains from attacking, the status quo is abandoned if and only if $\theta < \theta^s$, where θ^s is given by the unique solution to

$$\theta^s - \frac{\delta + \xi(1 - \gamma)\mathbf{P}(x \leq x_L^*|\theta^s)}{\tau} = 0. \quad (\text{A1})$$

If the large player attacks, then the status quo is abandoned if and only if $\theta < \theta^l$, where θ^l is given by the unique solution to

$$\theta^l - \frac{\delta + \xi(\gamma + (1 - \gamma)\mathbf{P}(x \leq x_L^*|\theta^l))}{\tau} = 0. \quad (\text{A2})$$

The indifference condition for the large player is given by

$$\mathbf{P}(\theta \leq \theta^s | y_L^*) = \nu. \quad (\text{A3})$$

The large player takes into account that the status quo is abandoned for $\theta \in [\theta^s, \theta^l]$ only if she attacks. However, the indifference condition for the small players takes into account that because of her noisy signal, the large player may nevertheless attack at some value $\theta \in [\theta^s, \theta^l]$, i.e.,

$$\frac{1}{2\varepsilon} \int_{x_L^* - \varepsilon}^{\theta^s} d\theta + \frac{1}{2\varepsilon} \int_{\theta^s}^{\min\{\theta^l, x_L^* + \varepsilon\}} \mathbf{P}(y \leq y_L^* | \theta) d\theta = \nu. \quad (\text{A4})$$

Changing the variables of integration allows to rewrite the latter equation as

$$\int_0^{n^s} dn + \int_{n^s}^{\min\{n^l, 1\}} G\left(\frac{\varepsilon}{\omega}(n^s - n) + (1 - \nu)\right) dn = \nu, \quad (\text{A5})$$

where $n^s = \min\left\{\frac{\theta^s - x_L^* + \varepsilon}{2\varepsilon}, 1\right\}$, $n^l = \min\left\{\frac{\theta^l - x_L^* + \varepsilon}{2\varepsilon}, 1\right\}$ and $G(z) = \min\{\max\{z, 0\}, 1\}$.

The equilibrium is given by the solution to the four equations [Eqs. \(A1\)-\(A3\)](#) and [Eq. \(A5\)](#). It is straightforward to establish that, given x_L^* , [Eqs. \(A1\)-\(A3\)](#) yield unique solutions for θ^s , θ^l and y_L^* . Thus, to show the existence of a unique threshold equilib-

rium, it is sufficient to show the existence of a unique x_L^* that solves Eq. (A5). Notice that the left-hand side of Eq. (A5) converges to unity for $x_L^* \rightarrow \theta^s - \varepsilon$ and to zero for $x_L^* \rightarrow \theta^l + \varepsilon$. Because the function is continuous in x_L^* , the intermediate value theorem implies the existence of at least one solution to Eq. (A5). The solution is unique because it follows from Eqs. (A1) and (A2) that $\frac{d\theta^s}{dx_L^*} \in (0, 1)$ and $\frac{d\theta^l}{dx_L^*} \in (0, 1)$ and thus $\frac{dn^s}{dx_L^*} < 0$ and $\frac{dn^l}{dx_L^*} < 0$. Therefore, the left-hand side of Eq. (A5) strictly decreases in x_L^* .

The solution depends on the ratio of the noise parameters ε/ω . We characterize the solution for the limit of vanishing noise, $\varepsilon \rightarrow 0$ and $\omega \rightarrow 0$ for the two extreme cases where either $\frac{\varepsilon}{\omega} \rightarrow \infty$ or $\frac{\varepsilon}{\omega} \rightarrow 0$.

For $\frac{\varepsilon}{\omega} \rightarrow \infty$, Eq. (A5) converges to

$$n^s = \nu.$$

Notice that

$$\mathbf{P}(x \leq x_L^* | \theta^s) = 1 - n^s.$$

Thus, we have

$$\theta^s = \frac{\delta + \xi(1 - \gamma)(1 - \nu)}{\tau}.$$

Moreover, in the limit for $\varepsilon \rightarrow 0$ and $\omega \rightarrow 0$, we also have $x_L^*, y_L^* \rightarrow \theta^s$.

Next, consider the case where $\frac{\varepsilon}{\omega} \rightarrow 0$. We have to distinguish two cases, namely $x_L^* < \theta^l$ and $x_L^* = \theta^l$. In the first case, Eq. (A5) yields

$$n^s + (1 - n^s)(1 - \nu) = \nu \Rightarrow 1 - n^s = \frac{1 - \nu}{\nu},$$

and therefore

$$x_L^*, y_L^* \rightarrow \theta^s = \frac{\delta + \xi \frac{(1 - \gamma)(1 - \nu)}{\nu}}{\tau}.$$

Since $\theta^l > x_L^*$ (by supposition), it follows that $\lim_{\varepsilon \rightarrow 0} n^l \rightarrow 1$ and $\theta^l = \frac{\delta + \xi \gamma}{\tau}$. Hence, this case obtains if and only if $\gamma > 1 - \nu$.

Finally, note that in the second case, $\gamma \leq (1 - \nu)$, all critical values converge to the threshold in the absence of a large player, i.e.,

$$x_L^*, y_L^*, \theta^s, \theta^l \rightarrow \frac{\delta + \xi(1 - \nu)}{\tau}.$$

□

Proof of Proposition 4. For the extensive proof, the reader is referred to [Sakovics and Steiner \(2012\)](#). Here, we merely show how the aggregate belief constraint can be used to calculate the critical threshold θ_S^* .

Let $\mathbf{x}^*$ denote the vector of critical signals x_g^* . The aggregate action can be expressed as

$$a(\mathbf{x}^*, \theta) = \sum_g m_g w_g G\left(\frac{x_g^* - \theta + \varepsilon}{2\varepsilon}\right),$$

where $G(z) = \min\{\max\{z, 0\}, 1\}$.

The players' strategic beliefs depend on the relative positions of the critical signals, i.e., $\Delta_g \equiv (x_g^* - x_1^*)/2\varepsilon$. We can write type g 's strategic belief as

$$\begin{aligned} A_g(\Delta, \ell) &= \mathbf{P}\left(a(\mathbf{x}^*, \theta) \leq \ell | x_g^*\right) \\ &= \mathbf{P}\left(\sum_h m_h w_h G\left(\frac{\Delta_h - \Delta_g + 2\varepsilon}{2\varepsilon}\right) \leq \ell\right) \end{aligned}$$

Lemma A1. *The weighted average strategic belief is uniform on $[0, 1]$: $\sum_g m_g w_g A_g(\Delta, \ell) = \ell$.*

Proof. See [Sakovics and Steiner \(2012\)](#). □

The probability that the threshold player from group g attaches to regime change is then given by

$$p_g = 1 - A_g\left(\Delta, \frac{\tau\theta_S^* - \delta}{\xi}\right).$$

Using the aggregate belief constraint from [Lemma A1](#),

$$\sum_g m_g w_g p_g = \sum_g m_g w_g \left(1 - A_g \left(\Delta, \frac{\tau \theta_S^* - \delta}{\xi} \right) \right) = 1 - \frac{\tau \theta_S^* - \delta}{\xi}.$$

For the threshold player from any group g , it must be that $p_g = v_g$. Thus, we obtain

$$\sum_g m_g w_g v_g = 1 - \frac{\tau \theta_S^* - \delta}{\xi} \Leftrightarrow \theta_S^* = \underline{\theta} \left(1 + \frac{\xi (1 - \sum_g m_g w_g v_g)}{\delta} \right).$$

□

Proof of [Proposition 5](#). This proof proceeds in two steps. First we show that, for each institution, j , the extreme beliefs satisfy $\hat{x} < \bar{x}$. And next we derive the joint equilibrium as the solution to two best-response correspondences for the players from the two institutions.

Extreme beliefs. Suppose that players of institution j anticipate that the players of institution $-j$ refrain from attacking. The critical mass condition for institution j is

$$\tau \theta_j^* - \xi a(x_j^*, \theta_j^*) - \delta = 0.$$

Solving this together with the players' indifference condition, we obtain that players of institution j use the critical threshold

$$\hat{x} \equiv \frac{\delta + (1 - \nu)\xi + \epsilon(1 - 2\nu)\tau}{\tau}.$$

Next, suppose that players of institution j anticipate that all players of institution $-j$ attack. The critical mass condition is

$$\tau \theta_j^* - \xi a(x_j^*, \theta_j^*) - \zeta - \delta = 0,$$

and solving this together with the indifference condition, we get the critical threshold

$$\tilde{x} \equiv \hat{x} + \frac{\zeta}{\tau}$$

for players of institution j . Clearly, $\hat{x} < \tilde{x}$ and under the assumption of homogeneous parameters across the two institution, the extreme beliefs are identical for the two groups.

Joint equilibrium. For institution j , we solve the general critical mass and indifference conditions – taking as given the critical threshold used by players of institution $-j$. The joint equilibrium is then given by the solution to

$$\begin{aligned} x_1^*(x_2) &= \frac{\zeta x_2 + 2\epsilon[(1-\nu)(\xi + \zeta) + \delta + \epsilon(1-2\nu)\tau]}{\zeta + 2\epsilon\tau} \\ x_2^*(x_1) &= \frac{\zeta x_1 + 2\epsilon[(1-\nu)(\xi + \zeta) + \delta + \epsilon(1-2\nu)\tau]}{\zeta + 2\epsilon\tau} \end{aligned}$$

Solving this out, we get that $x_1^*(x_2^*) = x_2^*(x_1^*) = x^{**} = \frac{\delta + (1-\nu)(\xi + \zeta) + \epsilon(1-2\nu)\tau}{\tau}$. In the limit $\epsilon \rightarrow 0$, we obtain our result that $x^{**} = \theta^{**} = \underline{\theta} \left(1 + \frac{(1-\nu)(\xi + \zeta)}{\delta}\right)$. $\square$

Proof of Proposition 6. First, we establish the ordering of the extreme beliefs. To this end, and building on the results presented in the proof of Proposition 5, we have that

$$\begin{aligned} \hat{x}_1 &= \frac{\delta_1 + (1-\nu)\xi}{\tau} + \epsilon(1-2\nu) \\ \tilde{x}_1 &= \hat{x}_1 + \frac{\zeta_1}{\tau} \\ \hat{x}_2 &= \frac{\delta_2 + (1-\nu)\xi}{\tau} + \epsilon(1-2\nu) \\ \tilde{x}_2 &= \hat{x}_2 + \frac{\zeta_2}{\tau} \end{aligned}$$

Since $\delta_2 > \delta_1$, it follows that $\hat{x}_1 < \hat{x}_2$. Next, we have that $\tilde{x}_1 > \hat{x}_2$ whenever

$$\zeta_1 > \delta_2 - \delta_1.$$

In addition, it is always true that $\tilde{x}_1 < \tilde{x}_2$. The ordering of the extreme beliefs satisfies

$$\hat{x}_1 < \hat{x}_2 < \tilde{x}_1 < \tilde{x}_2.$$

We, thus, have that the regime change across the two institutions are perfectly correlated.

Finally, the joint equilibrium is given by the solution to

$$\begin{aligned} x_1^*(x_2) &= \frac{\zeta_1 x_2 + 2\epsilon[(1-\nu)(\xi + \zeta_1) + \delta_1 + \epsilon(1-2\nu)\tau]}{\zeta_1 + 2\epsilon\tau} \\ x_2^*(x_1) &= \frac{\zeta_2 x_1 + 2\epsilon[(1-\nu)(\xi + \zeta_2) + \delta_2 + \epsilon(1-2\nu)\tau]}{\zeta_2 + 2\epsilon\tau} \end{aligned}$$

Let $x^{**} = x_1^*(x_2^*) = x_2^*(x_1^*)$. In the limit $\epsilon \rightarrow 0$, we have that $x^{**} = \theta^{**} = \alpha\theta_1^{**} + (1-\alpha)\theta_2^{**} + \frac{2(1-\nu)\zeta_1\zeta_2}{\tau(\zeta_1+\zeta_2)}$, where $\alpha = \frac{\zeta_2}{\zeta_1+\zeta_2}$.

□

B Private vs. Public Information: the Gaussian Model

B1 The Gaussian Regime Change Model

In [Sections 2](#) to [4](#), we assume a uniform prior for θ . This assumption is sufficient for rank-order beliefs to be uniform and it helps to guarantee uniqueness of the global game solution. If players share some public information that is common knowledge, then this information also affects the equilibrium and may interfere with equilibrium uniqueness.

In this appendix, we briefly introduce the Gaussian version of the regime change model without interlinkages. Because of its computational simplicity, the Gaussian regime change model has become the standard set-up to analyze the relationship between private and public information. In this setting the fundamental θ is drawn from a Gaussian distribution with a finite mean that can be interpreted as the realization of a public signal.

Unique threshold equilibrium in the Gaussian set-up. Suppose that $\Theta = \mathbb{R}$ and let

$$\theta \sim \mathcal{N}(\mu, 1/\alpha) \quad \text{and} \quad \varepsilon_i \sim \mathcal{N}(0, 1/\beta),$$

where μ is the prior mean of θ and α and β are the precisions (inverse variances) of the prior and the noise in players' signals respectively. Players share common knowledge about the distributions and its parameters. Notice that for $\alpha \rightarrow 0$, the prior is improper (diffuse). The assumption of an improper prior is an often-used default assumption in a large number of papers and it leads to the same results as the uniform prior assumption discussed in Section 2. In what follows, we consider a non-diffuse prior with $\alpha > 0$.²⁴

By standard Bayesian updating, the posterior distributions are

$$x_i | \theta \sim \mathcal{N}(\theta, 1/\beta) \quad \text{and} \quad \theta | x_i \sim \mathcal{N}\left(\frac{\alpha\mu + \beta x_i}{\alpha + \beta}, \frac{1}{\alpha + \beta}\right).$$

Let $\Phi(\cdot)$ ($\phi(\cdot)$) denote the cumulative distribution (probability density) function of the standard normal distribution. As in Section 2, the symmetric threshold equilibrium can be summarized by the tuple (x^*, θ^*) that solve the indifference and the critical mass condition. In the Gaussian set-up, the equilibrium conditions (IC) and (CM) be-

²⁴Note that the information structure above is equivalent to a setting where each player has an improper prior over $\mathbb{R}$ and, together with the private signal x_i , observes a public signal $y \sim \mathcal{N}(0, 1/\alpha)$.

come

$$\Phi\left(\sqrt{\alpha+\beta}\left(\theta^* - \frac{\alpha\mu + \beta x^*}{\alpha + \beta}\right)\right) - \nu = 0 \quad (\text{IC}')$$

and

$$\tau\theta^* - \delta - \xi\Phi\left(\sqrt{\beta}(x^* - \theta^*)\right) = 0. \quad (\text{CM}')$$

Unique threshold equilibrium. One can immediately see that, for given θ^* , there exists a unique value x^* that solves (IC'), i.e.,

$$x^* = \theta^* - \frac{\alpha}{\beta}(\mu - \theta^*) - \frac{\sqrt{\alpha+\beta}}{\beta}\Phi^{-1}(\nu) \quad (\text{B1})$$

Substituting x^* into (CM') yields an implicitly defined function for θ^* ,

$$G(\theta^*) \equiv \tau\theta^* - \delta - \xi\Phi\left(\frac{\alpha}{\sqrt{\beta}}(\theta^* - \mu) - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\nu)\right) = 0. \quad (\text{B2})$$

Proposition B1. [Morris & Shin (Gaussian model)] *If the following condition holds,*

$$\frac{\alpha}{\sqrt{\beta}} < \frac{\tau\sqrt{2\pi}}{\xi}, \quad (\text{B3})$$

where $1/\sqrt{2\pi} = \max_z \phi(z)$, then there exists a unique equilibrium of the Gaussian regime change game where each player i chooses action 1 if and only if $x_i \leq x^*$ and $R(a(x^*, \theta), \theta) < 0$ if and only if $\theta \leq \theta^*$. The thresholds (θ^*, x^*) solve Eqs. (B1) and (B2).

Proof of Proposition B1. A solution to Eq. (B2) exists because $G(\theta^*)$ is strictly continuous in θ^* , $G(\theta) < 0$ for all $\theta \leq \underline{\theta}$ and $G(\theta) > 0$ for all $\theta \geq \bar{\theta}$. Thus, by the intermediate value theorem, $G(\theta^*)$ admits at least one solution in $[\underline{\theta}, \bar{\theta}]$.

A necessary and sufficient condition for uniqueness is that at any solution θ^* , $G'(\theta^*) >$

0. The sufficient condition for uniqueness

$$\frac{\alpha}{\sqrt{\beta}} < \frac{\tau\sqrt{2\pi}}{\xi},$$

follows by differentiating the implicit function $G(\theta^*)$ with respect to θ^* .

□

Revisiting the uniqueness condition. Condition (B3) states that the variance of public information must be sufficiently large for the equilibrium to be unique. Notice that the condition is trivially satisfied in the limiting case of vanishing noise, i.e., if β becomes arbitrarily large (the equivalent to letting ε become vanishingly small). In this case, the unique solution to $G(\theta^*)$ becomes

$$\lim_{\beta \rightarrow \infty} \theta^* = \underline{\theta} \left(1 + \frac{\xi(1-\nu)}{\delta} \right),$$

which corresponds to the solution for the threshold in Proposition 2. The equivalence between the thresholds fails to hold outside of the limit because in the Gaussian case (with proper prior), aside from the limit, any solution to Eq. (B2) depends on the distribution functions and their parameters. To understand the intuition behind condition (B3), notice that whenever α increases relative to β , the weight attached by players to public information when forming their beliefs becomes larger compared to the weight attached to private information. Thus, the game moves closer to the complete information game with common knowledge where multiple equilibria occur. Hellwig (2002) provides a detailed treatment of the transition between uniqueness and multiplicity of equilibria.

Several papers have tested the robustness of condition (B3) in variants of the regime change model. Angeletos and Werning (2006) consider a two-stage game where players first trade an asset in a financial market and subsequently play the regime change

game. The asset's dividend is correlated with the fundamental θ . The price function aggregates players' private information about θ and is publicly observable. Thus, the endogenously determined asset price acts like a public signal. [Angeletos and Werning \(2006\)](#) show that in this case, more precise private information can bring the multiplicity problem back. The reason is the positive relation between the precisions of private and of endogenously generated public information: more precise private information leads, via the aggregation in the price function, to more precise public signal. Their results provide a note of caution for the use of global games with private and public information. The uniqueness conditions depend crucially on whether or not public information is exogenously given or endogenously generated and, for the latter case, it also matters how it is generated. Indeed, [Angeletos and Werning \(2006\)](#) show that multiplicity can obtain when the noise in private information is small, in sharp contrast to condition (B3).

[Kuhle \(2016\)](#) further qualifies condition (B3) by introducing heterogeneity into the players' priors. He assumes that each agent observes a prior with common precision but with a mean μ_i that is distributed according to $\mu_i \sim \mathcal{N}(\bar{\mu}, 1/\eta)$. All distributions are common knowledge. The equilibrium uniqueness condition becomes

$$\frac{\alpha}{\sqrt{\beta}} \left(\sqrt{\frac{\eta}{\eta + \frac{\alpha^2}{\beta}}} \right) < \frac{\tau \sqrt{2\pi}}{\xi}.$$

The latter condition implies that uniqueness obtains already for sufficiently dispersed priors, irrespective of the relative precision of private and public information. That is, uniqueness obtains if $\sqrt{\eta} < \tau \sqrt{2\pi}/\xi$. In light of the results by [Angeletos and Werning \(2006\)](#), the results by [Kuhle \(2016\)](#) suggest that a sufficiently dispersed prior is a more robust condition because it always implies uniqueness, notwithstanding the nature and source of public information.

B2 The Effect of Information Quality

An important question that can be addressed with the Gaussian framework concerns the effects of changes in information quality, i.e., changes in the precision of signals, on the equilibrium thresholds.²⁵ Metz (2002) shows that the effect of a higher β on θ^* depends itself on θ^* ,²⁶

$$\frac{\partial \theta^*}{\partial \beta} > 0 \quad \Leftrightarrow \quad \theta^* < \mu + \frac{\Phi^{-1}(\nu)}{\sqrt{\alpha + \beta}}.$$

Metz (2002) and Bannier and Heinemann (2005) offer a detailed discussion of this result. To understand the intuition behind it, note that θ^* is strictly decreasing in the public signal μ , while the right-hand side of the condition strictly increases in μ . Thus, there exists a critical μ_β , such that $\frac{\partial \theta^*}{\partial \beta} > 0 \Leftrightarrow \mu > \mu_\beta$. If the public signal is sufficiently high, then the fundamental θ tends to be high and the attack is likely to be unsuccessful (as $\theta > \theta^*$). Hence, players should refrain. An increase in the quality of private information leads players to place more weight on private than on public information so that they become more prone to attack (θ^* increases). The effect reverses whenever $\mu < \mu_\beta$.²⁷

Ahnert and Kakhbod (2016) extend the standard regime change model to endogenous information acquisition. They derive an amplification channel whereby private information choices amplify the adverse effect of deteriorating public information. They show that the above intensive margin effect of β on θ^* also holds for the extensive margin when more players become informed. That is, for a sufficiently high public signal, an increase in the share of informed investors leads to a higher θ^* .

²⁵We focus only on changes in the precision of private information β . For a further discussion of the effects of public information quality, see Metz (2002) or Morris and Shin (2003).

²⁶We consider marginal changes in β that do not affect the uniqueness condition (B3).

²⁷Iachan and Nenov (2015) study information quality in an extended model where the payoff parameters (B and C) are functions of θ . Their model includes the model studied here as a special case. In their generalized model, changes in information quality depend on the elasticities of the payoff parameters with respect to θ .

B3 Applications to Disclosure Regulation

The Gaussian model is a versatile framework to study the effects of information disclosure by banks and financial market participants. This literature departs from the original idea that the global game just constitutes a convenient device to select a unique equilibrium, but rather interprets the private and public signals literally as information being generated by a commonly known technology (or institution) where the precision of signals measures the quality of information.

A number of papers ([Heinemann, 2005](#); [Bannier, 2005](#); [Vives, 2014](#)) caution against the notion that more public disclosure (higher precision α) is generally beneficial. Rather, these papers show that the effect of more precise public information depends on the magnitude of the fundamentals (see also above in [Section B2](#)). Because the public signal acts as a coordinating device, making it more precise when the fundamentals are weak, makes players more prone to attack. Thus, more (less) disclosure is better when fundamentals are strong (weak). [Vives \(2014\)](#) also argues that the effect of prudential ratios, e.g., capital or liquidity ratios, depend on the degree of public information precision, implying that regulators should decide jointly on disclosure and prudential regulations.

[Bouvard et al. \(2015\)](#) consider the incentives of the regulator to disclose information in a financial system with different types of banks. They argue that the regulator should increase transparency about individual banks during a crisis in order to stabilize the banking system. However, they also show that the optimal disclosure policy is not time-consistent, implying that the regulator may create excess opacity and thereby raise the likelihood of a systemic crisis.

[Edmond \(2013, 2018\)](#) studies the incentives of a policymaker to manipulate information to prevent an attack from abandoning the status quo. He shows that signal-jamming can be effective when the precision of signals is sufficiently large. However,

if, at the same time, the number of signals increases, also the manipulation cost of the policymaker rise, in which case the attack is more likely to succeed. [Gao and Jiang \(2018\)](#) interpret signal manipulation as reporting discretion by a bank manager. They argue that banks that are more vulnerable to panic-based runs should enjoy a larger degree of reporting discretion to stave off a run. Interestingly, they find that reporting discretion can not only reduce the likelihood of illiquidity, but also prevent players to run on insolvent banks, implying that discretion can weaken market discipline.