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Calculation of Potential Vorticity on the ICON Grid

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Abstract

This article describes the calculation of Potential Vorticity (PV) on the ICON grid. The algorithm has been implemented into the model and the PV is available as output variable on model-, pressure-, height-, and isentropic surfaces.

Keywords: Potential Vorticity

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1. Introduction

Potential vorticity (PV) is defined according to Ertel (1942) by

$$\mathrm{PV} = \frac{1}{\rho}\left(\nabla \times \mathbf{v}_a\right) \cdot \nabla\theta,$$

with the air density ρ, the three-dimensional absolute wind $\mathbf{v_a}$ and the potential temperature θ. The absolute wind is definded as the sum of the wind relative to the rotating earth, $\mathbf{v} = (u, v, w)$ and the velocity of the rotation itself, $\mathbf{v}_r = \mathbf{\Omega} \times \mathbf{r}$. With the Coriolis parameter $\mathbf{f} = \nabla \times (\mathbf{\Omega} \times \mathbf{r}) = 2\mathbf{\Omega}$ the PV can be written as

$$\mathrm{PV} = \frac{1}{\rho}\left(\nabla \times \mathbf{v} + \mathbf{f}\right) \cdot \nabla\theta. \tag{1}$$

To derive a numerical method to calculate PV on the ICON grid, equation (1) will be transformed to spherical coordinates, locally rotated to the edge-coordinate system of the grid and finally transformed to a generalized vertical coordinate system. The resulting equation can then be used for discretisation with already implemented standard ICON methods.

2. Spherical coordinates

Following Vallis (2006), the curl of a vector field $\mathbf{B}$ in geographic spherical coordinates can be written as

$$\begin{aligned}\nabla \times \mathbf{B} = &\left(\partial_y B_z - \partial_z B_y - \frac{B_y}{r_e + z}\right)\mathbf{x} \\ &+ \left(\partial_z B_x - \partial_x B_z + \frac{B_x}{r_e + z}\right)\mathbf{y} \\ &+ \left(\partial_x B_y - \partial_y B_x + \frac{\tan\vartheta}{r_e + z} B_x\right)\mathbf{z},\end{aligned}$$

where the latitude ϑ, the longitude ϕ and the distance to the earth midpoint r have already been replaced with the usual local-cartesian coordinates $x = r\phi\cos\vartheta$, $y = r\vartheta$ and $z = r - r_e$, with the earth radius r_e. With this, the PV on the sphere is given by

$$\rho\,\mathrm{PV} = \begin{pmatrix} \partial_y w - \partial_z v - \dfrac{v}{r_e + z} + f_x \\ \partial_z u - \partial_x w + \dfrac{u}{r_e + z} + f_y \\ \partial_x v - \partial_y u + \dfrac{u\tan\vartheta}{r_e + z} + f_z \end{pmatrix} \cdot \begin{pmatrix} \partial_x \theta \\ \partial_y \theta \\ \partial_z \theta \end{pmatrix}. \tag{2}$$

3. Local rotation to edge coordinate system

At the edge midpoints on the ICON grid, an orthogonal coordinate system is defined by the vector $\mathbf{n}$, pointing normal to the edge, the vector $\mathbf{t}$, pointing tangential to the edge and the vertical direction $\mathbf{k}$, with $\mathbf{t} \times \mathbf{n} = \mathbf{k}$ and $\mathbf{k}$ pointing away from the earth midpoint. The horizontal part of the scalar product in (2) is invariant under a rotation of the coordinate system around the z-axis. In addition, the vorticity can equally be expressed in such a rotated coordinate system. Thus with $y \to n$ and $x \to t$ the PV equation can be rewritten as

$$\rho\,\mathrm{PV} = \begin{pmatrix} \partial_n w - \partial_z v_n - \frac{v_n}{r_e+z} + f_t \\ \partial_z v_t - \partial_t w + \frac{v_t}{r_e+z} + f_n \\ \partial_t v_n - \partial_n v_t + \frac{u\tan\vartheta}{r_e+z} + f_z \end{pmatrix} \cdot \begin{pmatrix} \partial_t \theta \\ \partial_n \theta \\ \partial_z \theta \end{pmatrix}.$$

4. Generalized vertical coordinate

The final step is to transform PV to the models vertical coordinate, $h : (t, n, \eta) \to (t, n, z)$, with $z = z(t, n, \eta)$. The Jacobian D of this transformation reads

$$Dh = \frac{\partial(t, n, z)}{\partial(t, n, \eta)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{\partial z}{\partial t} & \frac{\partial z}{\partial n} & \frac{\partial z}{\partial \eta} \end{pmatrix}.$$

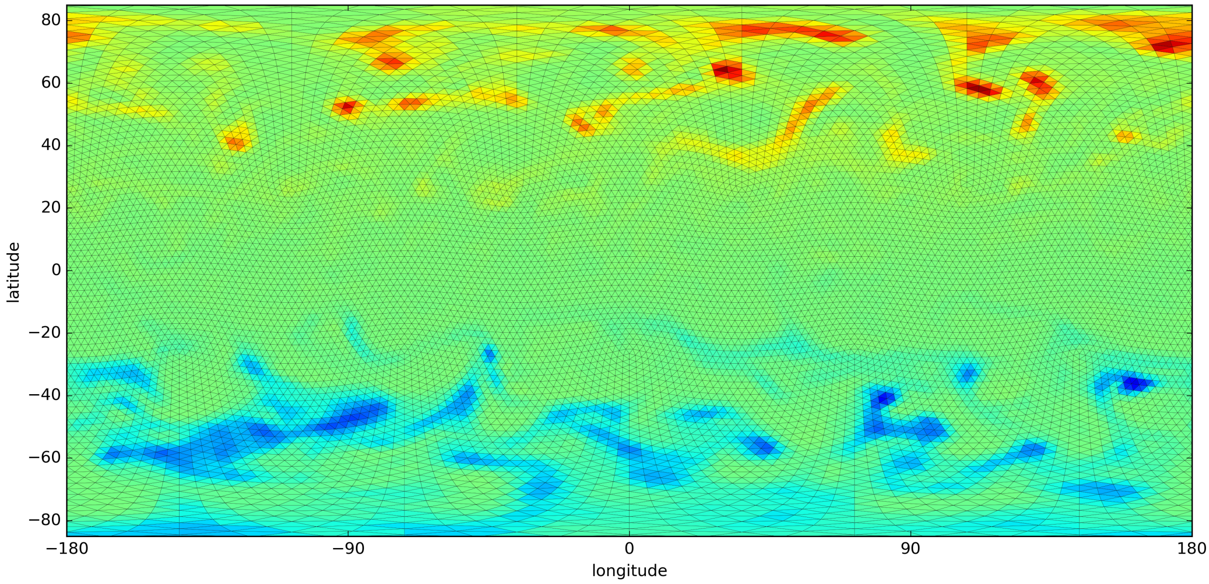

Figure 1: Example of a PV field on 300 hPa on a native R2B4-grid.

Let now g_η be a scalar field, expressed in the (t, n, η)-coordinates and g_z be the same scalar field, expressed in (t, n, z)-coordinates. Then $g_\eta = g_z \circ h$ and

$$Dg_\eta = (Dg_z) \cdot (Dh),$$

which leads to

$$\begin{aligned}
\frac{\partial g_z}{\partial t} &= \frac{\partial g_\eta}{\partial t} - \frac{\partial g_\eta}{\partial \eta}\frac{\partial z}{\partial t}\left(\frac{\partial z}{\partial \eta}\right)^{-1} \\
\frac{\partial g_z}{\partial n} &= \frac{\partial g_\eta}{\partial n} - \frac{\partial g_\eta}{\partial \eta}\frac{\partial z}{\partial n}\left(\frac{\partial z}{\partial \eta}\right)^{-1} \\
\frac{\partial g_z}{\partial z} &= \frac{\partial g_\eta}{\partial \eta}\left(\frac{\partial z}{\partial \eta}\right)^{-1}.
\end{aligned}$$

With the shortcut $\gamma = \partial z / \partial \eta$ this gives us the equation for PV in the model coordinate system:

$$\rho\,\mathrm{PV} = \begin{pmatrix} \partial_n w + \frac{\partial_n z}{\gamma}\partial_\eta w + \frac{1}{\gamma}\partial_\eta v_n - \frac{v_n}{r_e+z} + f_t \\ -\frac{1}{\gamma}\partial_\eta v_t - \partial_t w - \frac{\partial_t z}{\gamma}\partial_\eta w + \frac{v_t}{r_e+z} + f_n \\ \partial_t v_n - \partial_n v_t + \frac{u \tan\vartheta}{r_e+z} + \frac{\partial_t z}{\gamma}\partial_\eta v_n - \frac{\partial_n z}{\gamma}\partial_\eta v_t + f_z \end{pmatrix} \cdot \begin{pmatrix} \partial_t\theta + \frac{\partial_t z}{\gamma}\partial_\eta\theta \\ \partial_n\theta + \frac{\partial_n z}{\gamma}\partial_\eta\theta \\ -\frac{1}{\gamma}\partial_\eta\theta \end{pmatrix}.$$

Note that the wind vector (v_t, v_n, w) in the model is still expressed in the original (t, n, z)-system, which implies that the wind components have to be treated as scalar fields.

5. Implementation

The calculation of the PV is done in the subroutine `compute_field_pv` which is defined in the file `mo_util_phys.f90`. Regardless of the setting for the dynamical core the PV calculation makes use of the shallow atmosphere approximation, which implies $r_e + z \approx r_e$ and $f_t = f_n = 0$. The relative vorticity (p_diag%omega_z) is computed in ICON with Stokes theorem as a line integral of v_n along the edges of a dual cell and is thus given at the vertices of the primal cell (Bonaventura and Ringler, 2005). We will use this vorticity directly instead of the expression given above. Note, that the vorticity computed in this way also includes the $\tan\theta$-term, but not the metric terms, i.e.,

$$\texttt{p\_diag\%omega\_z} = \partial_t v_n - \partial_n v_t + \frac{u \tan\vartheta}{r_e}.$$

The calculation of PV is performed at the edge midpoints. Derivatives in t- and n-direction are provided by the model procedures grad_fd_tang and grad_fd_norm, for which the field must be defined at vertices and centers, respectively. If a field is not defined at these locations it will be interpolated there before. Vertical derivatives of full-level variables will be computed as $\partial_\eta f_i = (f_{i+1} - f_{i-1})/2$ and as one-side derivatives at the vertical boundaries. The vertical wind w is defined at half levels, thus its vertical derivative equals $\partial_\eta w_i = w_{i+1/2} - w_{i-1/2}$ and its interpolation to full levels is given by $w_i = (w_{i+1/2} + w_{i-1/2})/2$. After being calculated at the edge midpoints, ρPV is interpolated back to the cell centers and is finally normalized by the air density p_prog%rho, i.e. the density of the air including moisture and water condensates.

The output of the PV can be on the original model levels or it can be interpolated to z-, p- and θ-levels. The variable name is 'pv' and output is in SI-units. An example namelist for PV-output on the 300 hPa-isobar can be found below, the according example PV-field is shown in Figure 1. Here, the output is done on the native ICON-grid, but horizontal remapping is also possible.

```
&output_nml
  ...
  pl_varlist        =  'pv',
  p_levels          =  30000,
\
```

References

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